Multi-objective linear optimization for strategic planning of shared autonomous vehicle operation and infrastructure design

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1 INTRODUCTION

Shared autonomous vehicle (SAV) systems can be an efficient transportation mode in the future (Fagnant and Kockelman, 2015). In an SAV system, autonomous vehicles shared by the society will transport travelers by using optimized routes and/or ridesharing matching. Thus, it will decrease number of vehicles and parking lots in a city without sacrificing travelers utility.

Design of SAV systems involves various types of problems. The notable examples are vehicle routing problem with pickup and delivery with time windows (VRPPDTW) (Mahmoudi and Zhou, 2016; Aiko *et al.*, 2017), dynamic ridesharing matching (Regue *et al.*, 2016; Aiko *et al.*, 2017; Thaithatkul *et al.*, 2019), fleet size optimization (Vazifeh *et al.*, 2018), road network design and autonomous vehicle lane deployment (Chen *et al.*, 2016), and parking space allocation. In the previous studies, these problems were often solved separately. Furthermore, they were often formulated by using computationally costly frameworks such as mixed integer programming.

The importance of trade-off relations among performance indexes of SAV systems has been noted, especially in strategic levels. For example, an SAV system could be designed to minimize either user-side cost (e.g., passenger travel time), system-side cost (e.g., operational cost), or social-side cost (e.g., environmental cost); these cases may have completely different system design and cost allocation. Such trade-off relations can be explicitly investigated by using the framework of multi-objective optimization problems (MOOP); however, to the authors' knowledge, application of MOOP to SAV system modeling is very limited.

This study proposes a unified MOOP for aggregated versions of VRPPDTW, dynamic ridesharing matching, fleet size optimization, road network design, and parking space allocation of SAV systems based on dynamic traffic assignment (DTA). The proposed problem is formulated as linear programming, making it very easy to solve. Meanwhile, it approximates vehicles and travelers as continuum flow. These features will be useful for strategic optimization of SAV operation and infrastructure design.

2 FORMULATION

The proposed problem is based on the maximal flow problem with a time-expanded network. Specifically, a road network is modeled as a time-expanded network shown in Fig. 1. Then, vehicles and passenger flows, link capacity, and node capacity in the expanded network are optimized under given time-dependent origin-destination (OD) matrix of passengers.

The basic idea of the problem is as follows. Let x^t_{ij} be the total number of SAVs that travel from node i to j at

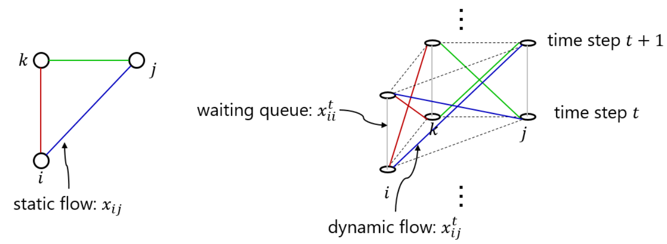


Figure 1: Left: standard network. Right: time-expanded network for dynamic traffic assignment

time step t. Let $y_{s,ij}^{k,t}$ be the total number of travelers with certain properties (destination s, departure time k) that travel from node i to j at time step t. Since travelers need to ride SAVs to travel, condition $\sum_{s,k} y_{s,ij}^{k,t} \leq \rho x_{ij}^t$ must be satisfied, where ρ is a given passenger capacity of a SAV. Furthermore, x_{ij}^t must be smaller than the link capacity. Our objective is to find the most efficient $y_{s,ij}^{k,t}$ and x_{ij}^t and other decision variables under proper constraints including the passenger capacity and the link capacity.

The problem is formulated follows:

$$\text{[SOSAV]} \qquad \min\,(T,\ D,\ N,\ C) \tag{1}$$

such that

$$\sum_{ij,s,t,k} t_{ij} y_{s,ij}^{k,t} = T \qquad \text{(total travel time)} \tag{2}$$

$$\sum_{ij,i\neq j} d_{ij} x_{ij}^t = D \qquad \text{(total distance traveled)} \tag{3}$$

$$\sum_i x_{0i}^0 = N \qquad \text{(fleet size)} \tag{4}$$

$$\sum_{ij} c_{ij}(\mu_{ij} - \mu_{ij}^{\min}) + \sum_i c_i(\kappa_i - \kappa_i^{\min}) = C \qquad \text{(total construction cost)} \tag{5}$$

$$\sum_j x_{ji}^{t-1} - \sum_j x_{ij}^t = 0 \qquad \forall i, t \in (0, t_{\max}) \qquad \text{(meaning: vehicle conservation)} \tag{6}$$

$$\sum_j y_{s,ji}^{k,t-1} - \sum_j y_{s,ij}^{k,t} + y_{s,0i}^{k,t} - y_{s,i0}^{k,t} = 0 \qquad \forall i, s, k, t \in T_k \qquad \text{(passenger conservation)} \tag{7}$$

$$\sum_{s,k} y_{s,ij}^{k,t} \leq \rho x_{ij}^t \qquad \forall ij, i \neq j, t \qquad \text{(passenger capacity of SAV)} \tag{8}$$

$$x_{ij}^t \leq \mu_{ij} \qquad \forall ij, i \neq j, t \qquad \text{(traffic capacity of link)} \tag{9}$$

$$x_{ii}^t \leq \kappa_i \qquad \forall i, t \qquad \text{(parking capacity of node)} \tag{10}$$

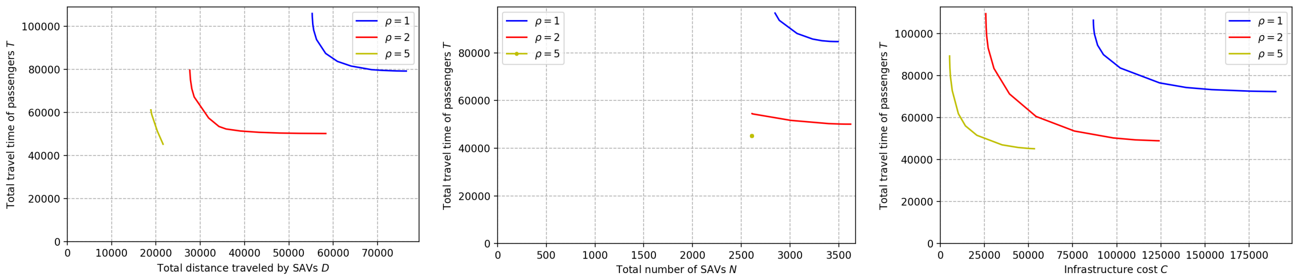
$$y_{s,0r}^{k,k} = M_{rs}^k \qquad \forall rs, k \qquad \text{(passenger departure)} \tag{11}$$

$$\sum_{t\in[k,k+d_{\max}]} y_{s,s0}^{k,t} = \sum_r M_{rs}^k \qquad \forall s, k \qquad \text{(passenger arrival)} \tag{12}$$

$$\sum_{ij} c_{ij}\mu_{ij} + \sum_i c_i \kappa_i \leq C \qquad \text{(construction budget)} \tag{13}$$

and some technical constraints such as non-negative constraints (omitted due to the space limitation), where T denotes the total travel time of passengers, D denotes the total distance traveled by SAVs, N denotes the total number of SAVs, C denotes the total infrastructure cost, μ_{ij} denotes the traffic capacity of link ij, κ_i denotes the parking capacity of node i, M_{rs}^k denotes the time-dependent OD matrix, $d_{\max}$ denotes the maximum allowable delay for travelers, $t_{\max}$ denotes the final time step, c_{ij} denotes the unit cost for traffic capacity expansion of link ij, c_i denotes the unit cost for parking capacity expansion of node i, α and β denote weight parameters, and $T_k = \{t \in (0, t_{\max}) \cap (k, k + d_{\max}]\}$.

The main decision variables are x_{ij}^t (corresponds to VRPPDTW with ridesharing), $y_{s,ij}^{k,t}$ (VRPPDTW with ridesharing), N (fleet size problem), μ_{ij} (link construction or SAV lane deployment problem), and κ_i (parking space



(a) (T, D) domain.

(b) (T, N) domain.

(c) (T, C) domain.

Figure 2: Pareto frontiers.

allocation problem). Notice that these variables are in linear relationship in the problem. Thus, this is linear programming. The computation time is polynomial to the number of links and time steps.

The MOOP finds the Pareto frontier in (T, D, N, C) domain, in which any of the objective function values cannot be decreased without increasing the other(s). A decision-maker would select one of the solutions from the Pareto frontier by considering the society's policies and trade-off relations among the objective functions.

This problem can be considered as a point queue-based DTA with vehicle queuing on nodes with limited queue length. The queue size on a node is constrained by (10); x_{ii}^t can be interpreted as the sum of parking vehicles and waiting vehicles on curbside. In fact, the problem can be considered as a variant of DTA-based optimal evacuation problem of Kuwahara *et al.* (2017). The limitation of this problem is that it only computes aggregated link flows; therefore, path flows and travel routes of individual travelers cannot be identified uniquely.

Problem [SOSAV] has useful policy implication on ridesharing. It is mathematically guaranteed that the optimal values of T, D, N, and C of [SOSAV] are simultaneously and monotonically non-increasing by increasing the passenger capacity ρ. Thus, ridesharing in [SOSAV] is always beneficial to average travelers as well as vehicle operators, road authorities, and the environment in the proposed model. The proof is omitted from this abstract.

3 NUMERICAL EXAMPLES

To investigate the quantitative behaviors of the proposed model under somewhat realistic conditions, a numerical experiment with actual travel data from New York City (NYC) was conducted. The passenger demand was generated from the NYC taxi data (Taxi and Limousine Commission, 2020). The travel records of taxis from 8:00 to 9:00 on 2019-04-01 (Monday) in Manhattan were extracted. We assumed that these travel records were equivalent to travel requests by SAV users in this area. The total number of passengers was 17,998. Then, the travel requests were aggregated to the time-dependent OD matrix M_{rs}^k with a 5 min time discretization width and a 30 min departure time aggregation width. We used a zone-based network for DTA.

Pareto frontiers in two-dimensional domains are illustrated in Fig. 2. In these plots, the relation between two objective values (i.e., T and one of the others) is shown where the rest of the objective values are fixed to certain values (i.e., $D = 60000$, $N = 3500$, $C = 100000$). According to the figure, the trade-off relation between the objectives were clearly found. Furthermore, by comparing the no-ridesharing cases ($\rho = 1$) to the two-person ridesharing cases ($\rho = 2$) or five-person ridesharing cases ($\rho = 5$), the efficiency of ridesharing was evident.

Fig. 3 shows the spatial distributions of SAV flow (i.e., $\sum_t x_{ij}^t$ where $i \neq j$) in several Pareto efficient solutions.

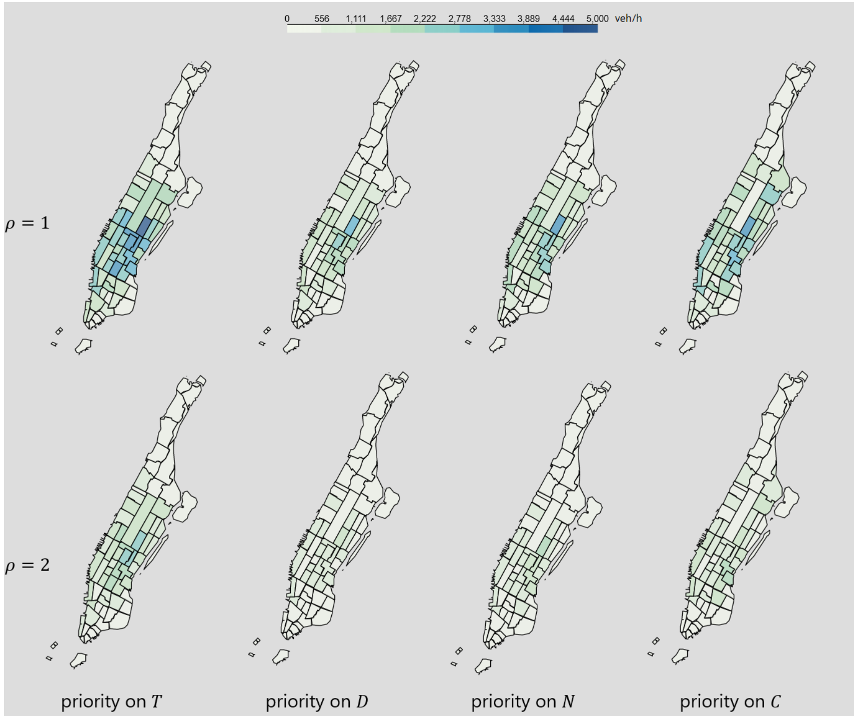


Figure 3: Spatial distribution of SAV flow under various conditions.

According to the figure, when the minimization of T was prioritized (left column), high SAV flow would be observed to enable quick transportation of travelers. This high flow included travel of empty vehicles. Contrary, when the other objectives were prioritized, the SAV flow would be significantly reduced. The introduction of ridesharing ($\rho = 2$, lower row) also reduced the SAV flow.

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