Calibration-free traffic state estimation method using single detector and connected vehicles with Kalman filtering and RTS smoothing

Toru Seo

***Abstract*— Traffic state estimation (TSE), which reconstructs complete traffic states from partial observation data, is an essential component in intelligent transportation systems. In this study, a novel traffic state estimation method using connected vehicles and a single detector based on Kalman filtering and Rauch–Tung–Striebel (RTS) smoothing is proposed. To the author's knowledge, while filtering is common approach for TSE, smoothing has not been employed to TSE in the literature. The important features of the proposed method are twofold. First, thanks to RTS smoothing, it can estimate accurate traffic state using a single detector, and it does not require detectors in every entries and exits of a road section. In addition, the estimation accuracy is not significantly sensitive to detector location. Second, it does not require parameter calibration thanks to the method's data-driven nature. These features will make the method flexibly applicable for practical conditions. Estimation accuracy of the proposed method was empirically evaluated by using actual vehicle trajectories data, and the effectiveness of the above two features was confirmed.**

I. Introduction

Traffic state estimation (TSE) [1], [2] is an essential component in intelligent transportation systems. The most popular approach for TSE is data assimilation that uses *filtering* (e.g., *Kalman filter* and its variants) [2]. In data assimilation approaches, heterogeneous traffic data collected from different sources are combined by a traffic flow model to reconstruct unobserved traffic states.

Implementation of TSE methods to actual traffic is still challenging. The notable reasons would be twofold: difficulty of parameter calibration and need of traffic detectors. Most of TSE methods depend various types of parameters whose calibration is not trivial or inessential. The notable examples is fundamental diagram (also known as flow–density relation) parameters such as free-flow speed and traffic capacity. In this study, a calibration-free TSE method is developed by data-driven approaches employed by the existing studies [3]–[7].

Traffic detectors (e.g., loop detectors) generally provides useful information for TSE, such as traffic counts and occupancy. However, due to their high installation and maintenance cost, their practical availability is limited. In order to enable detector-less TSE, many recent studies have proposed use of *connected vehicles (CVs)* [2], [6], [8]–[10]. CVs can be used to collect traffic speed information, and they are ubiquitously available in entire road networks in these days.

T. Seo is with Department of Civil Engineering, the University of Tokyo. Email: seo@civil.t.u-tokyo.ac.jp (T. Seo). The complete trajectory data were provided by Hanshin Expressway Co., Ltd. This work was partly supported by KAKENHI Grant-in-Aid for Scientific Research (16K18164, 19H02268) of Japan Society for the Promotion of Science.

However, multiple number of detectors are still required for accurate TSE by the CV-based existing methods [2], [6], [8]–[10]. For example, Bekiaris-Liberis et al. [6], [7] have proposed novel TSE methods based on Kalman filter using CVs and small number of detectors. They have mathematically show that detectors are required at every entries and exits in order to guarantee the theoretical observability of traffic system. Furthermore, they have empirically shown that TSE accuracy can be relatively poor if detector configuration does not satisfy the aforementioned condition. These results can be considered as a limitation of filtering-based TSE.

In order to overcome this limitation, this study proposes combined use of *smoothing* and filtering, which has not been utilized for TSE to the author's knowledge. Smoothing is a type of data assimilation, in which a state of a certain time is estimated by using data collected in the future. We will show that smoothing-based estimation is particularly useful for TSE with single detector because of the nature of information propagation in traffic flow. Note that smoothing is not suitable for real-time or on-line TSE as it uses the future information; however, it would be useful for number of practical purposes such as ex-post or off-line evaluation of traffic flow performance.

The aim of this research is to develop a filtering and smoothing-based TSE method using single detector and CV data, and to investigate the empirical performance of the proposed method by using actual vehicle trajectories data. Because of its simple mechanism, the proposed method can be expected to be flexibly applicable for practical conditions.

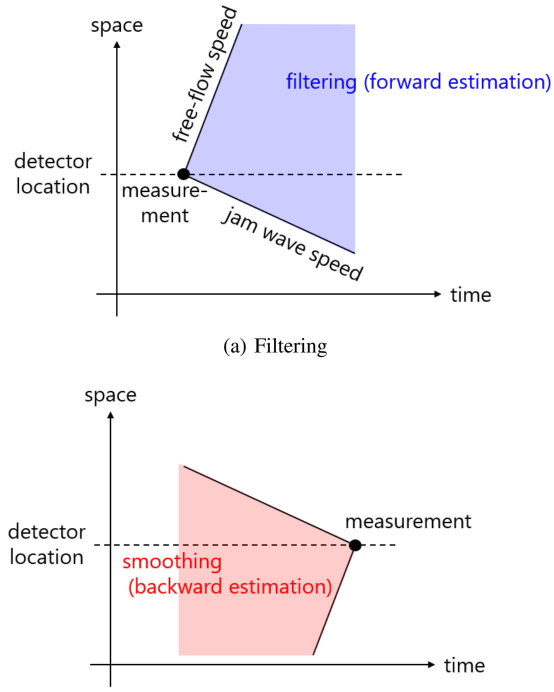
II. Method

A. Kalman filter and RTS smoother

Kalman filter [11] and its variants (e.g., particle filters) have been widely used as TSE methods [2]. In general, it estimates traffic state at time step T (denoted by $\boldsymbol{x}_T$) based on traffic data collected from time step 1 to T (denoted as $\boldsymbol{y}_{1:T}$) by maximizing the probability $p(\boldsymbol{x}_T|\boldsymbol{y}_{1:T})$. Thus, this approach is useful for on-line or real-time traffic management.

However, for the purpose of off-line traffic management (e.g., ex-post evaluation of traffic operation performance, collecting historical congestion statistics), Kalman filter is not optimal in the sense that it does not necessarily maximize $p(\boldsymbol{x}_t|\boldsymbol{y}_{1:T})$ where $1 \leq t < T$. In general, it is beneficial if one can estimate $\boldsymbol{x}_t$ that maximizes $p(\boldsymbol{x}_t|\boldsymbol{y}_{0:T})$; this is referred to as *maximum a posterior (MAP)* solution.

The combination of filtering and smoothing computes MAP solution. *Rauch–Tung–Striebel (RTS) smoother* [12]



(a) Filtering

(b) Smoothing

Fig. 1: Detector location and its information propagation area in filtering- or smoothing-based estimation.

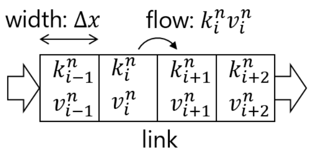


Fig. 2: Cell-based discretization of traffic on a link.

is one of the typical methods for smoothing, applicable for linear Gaussian state-space model. Despite this useful fact, the RTS smoother or its variants have not been utilized for TSE problems in the literature to the author's knowledge.

By combining filtering and smoothing, a single detector will become useful to estimate traffic state regardless of its location, because of the nature of traffic flow (i.e., Kinematic Wave theory [13]–[15]). Specifically, filtering will be useful to estimate forward moving waves of traffic, whereas smoothing will be useful to estimate backward moving one Fig. 1a. Smoothing is not applicable for real-time estimation; however, it would be still useful for some practical purposes such as ex-post evaluation.

B. System and observation models

We consider traffic on a link without intermediate entries or exits. We discretize the link by time and space widths Δt and Δx as in usual cell-based traffic flow models (Fig. 2). Our objective is to estimate k_i^n, the density in cell i at time step n, based on $\hat{k}_i^n$, the observed density by detector(s), and v_i^n, the observed traffic speed by CVs.

In this method, system's dynamics and observation is modeled by the following state-space model:

$$\boldsymbol{x}_n = \boldsymbol{f}_n(\boldsymbol{x}_{n-1}), \tag{1}$$

$$\boldsymbol{y}_n = \boldsymbol{h}_n(\boldsymbol{x}_n), \tag{2}$$

where $\boldsymbol{x}_n$ denotes the system's state at time step n, $\boldsymbol{y}_n$ denotes the observation at time step n, $\boldsymbol{f}_n$ denotes the system model at time step n, and $\boldsymbol{h}_n$ denotes the observation model at time step n.

The system's state is specified as a traffic density in each road segment:

$$\boldsymbol{x}_n = (\dots,\ k_i^n,\ \dots). \tag{3}$$

The observation is also specified as an observed traffic density in road segments with detectors:

$$\boldsymbol{y}_n = (\dots,\ \hat{k}_i^n,\ \dots). \tag{4}$$

Note that if a detector measures the traffic count q_i^n instead of the density, the density can be obtained by combining the traffic count and CV speed: $\hat{k}_i^n = q_i^n / v_i^n$.

The conservation law is employed as the system model as in the existing studies [3]–[7]. It can be expresses as the following partial differential equation

$$\frac{\partial k}{\partial t} + \frac{\partial kv}{\partial x} = 0, \tag{5}$$

where $k(t,x)$ denotes the density at time t and location x, and $v(t,x)$ denotes the average speed at time t and location x. We assume that CVs are ubiquitously available in the road segment as in the existing studies [3]–[7]. Thus, v is known regardless of t and x. Then, Eq. (5) can be numerically computed by the upwind scheme as follows:

$$k_i^{n+1} = k_i^n + \frac{\Delta t}{\Delta x}\left(k_{i-1}^n v_{i-1}^n - k_i^n v_i^n\right), \quad \forall i > 0 \tag{6}$$

with parameters $(\dots,\ v_i^n,\ \dots)$, which is directly derived from CV data. The upstream boundary (cell 0) is assumed as

$$k_0^{n+1} = k_0^n. \tag{7}$$

The observation model is simply formulated as follows

$$k_i^n = \hat{k}_i^n, \qquad \forall i \text{ where detector is installed.} \tag{8}$$

Finally, by adding simple white noise terms, Eq. (3)–(8) can be expressed as the following linear state-space model in a matrix form without losing generality:

$$\boldsymbol{x}_n = F_n \boldsymbol{x}_{n-1} + \boldsymbol{\nu}_n, \tag{9a}$$

$$\boldsymbol{y}_n = H_n \boldsymbol{x}_n + \boldsymbol{\omega}_n, \tag{9b}$$

where F_n denotes the system model, H_n denotes the observation model,$\boldsymbol{\nu}_n$ denotes the system noise such that $\boldsymbol{\nu}_n \sim \mathcal{N}(0, Q_n)$, and $\boldsymbol{\omega}_n$ denotes the additive observation noise such that $\boldsymbol{\omega}_n \sim \mathcal{N}(0, R_n)$. The variance-covariance matrices Q_n and R_n are assumed as diagonal matrices with constant values denoted by σ_Q^2 and σ_R^2, respectively.

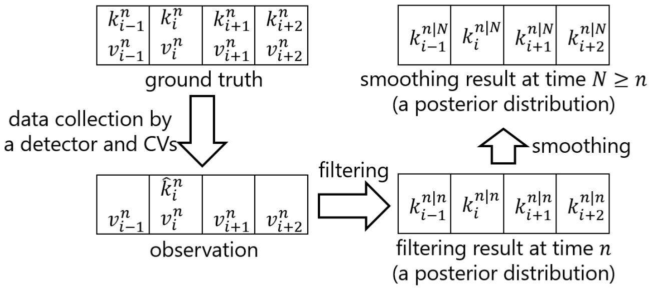


Fig. 3: TSE procedure by filtering and smoothing.

C. Filtering and Smoothing

By applying Kalman filtering and RST smoothing to Eq. (9), we can estimate $\boldsymbol{x}_n$ such that $p(\boldsymbol{x}_n|\boldsymbol{y}_{1:N})$ is maximized where N denotes the final time step. In this procedure, Kalman filtering computes a posterior distribution of the state, whereas RTS smoothing computes MAP distribution. The algorithm can be expressed as follows (due to the space limitation, the minimum necessary information is provided here. For the details, see the literature [11], [12], [16]).

Step 1 (Kalman Filtering): Set $n = 1$. Define the initial conditions of time step 0.

Step 1.1: Computes a prior distribution of time step n:

$$\boldsymbol{x}_{n|n-1} = F_n \boldsymbol{x}_{n-1|n-1} \tag{10}$$

$$V_{n|n-1} = F_n V_{n-1|n-1} F_n' + Q_n \tag{11}$$

Step 1.2: Computes a posterior distribution of time step n:

$$K_n = V_{n|n-1} H_n' (H_n V_{n|n-1} H_n' + R_n)^{-1} \tag{12}$$

$$\boldsymbol{x}_{n|n} = \boldsymbol{x}_{n|n-1} + K_n(\boldsymbol{y}_n - H_n \boldsymbol{x}_{n|n-1}) \tag{13}$$

$$V_{n|n} = V_{n|n-1} - K_n H_n V_{n|n-1} \tag{14}$$

Step 1.2: If $n = N$, go to Step 2. Otherwise set $n := n + 1$ and go to Step 1.1.

Step 2 (RTS smoothing): Set $n := N - 1$.

Step 2.1: Computes MAP distribution of time step n:

$$A_n = V_{n|n} F_{n+1}' V_{n+1|n}^{-1} \tag{15}$$

$$\boldsymbol{x}_{n|N} = \boldsymbol{x}_{n|n} + A_n(\boldsymbol{x}_{n+1|N} - \boldsymbol{x}_{n+1|n}) \tag{16}$$

$$V_{n|N} = V_{n|n} + A_n(V_{n+1|N} - V_{n+1|n}) A_n' \tag{17}$$

Step 2.2: If $n = 0$, halt the algorithm. Otherwise set $n := n - 1$ and go to Step 2.1.

The notation in the algorithm is as follows: $\boldsymbol{x}_{n|n-1}$ denotes the mean of a prior distribution of $\boldsymbol{x}_n$, $V_{n|n-1}$ denotes the variance-covariance matrix of a prior distribution of $\boldsymbol{x}_n$, $\boldsymbol{x}_{n|n}$ denotes the mean of a posterior distribution of $\boldsymbol{x}_n$, $V_{n|n}$ denotes the variance-covariance matrix of a posterior distribution of $\boldsymbol{x}_n$, K_n denotes the Kalman gain, $\boldsymbol{x}_{n|N}$ denotes the mean of MAP distribution of $\boldsymbol{x}_n$, and $V_{n|N}$ denotes the variance-covariance matrix of MAP distribution of $\boldsymbol{x}_n$. The procedure of the proposed method is illustrated in Fig. 3.

III. Evaluation

A. Data

For the evaluation purpose, traffic data collected at an urban expressway is used. The data is termed ZTD data [17]. It is a complete vehicle trajectories dataset collected by video cameras at 2 km length highway segment in Osaka, Japan for 1 hour duration; to the author's knowledge, this is the most large-scale complete vehicle trajectories dataset. Thus, it provides reliable, large-scale ground truth traffic states and simulated traffic detector and CV data. The ground truth traffic density is shown in Fig. 4a as a time–space diagram. Various traffic conditions, such as free-flowing traffic, congested traffic, series of stop-and-go waves, were observed. For the technical details on ZTD data, see [17], [18].

B. Results

The values of parameters are set as follows: $\Delta t = 4$ (s), $\Delta x = 100$ (m), $\sigma_Q = 0.01$ (veh/m), and $\sigma_R = 0.001$ (veh/m),

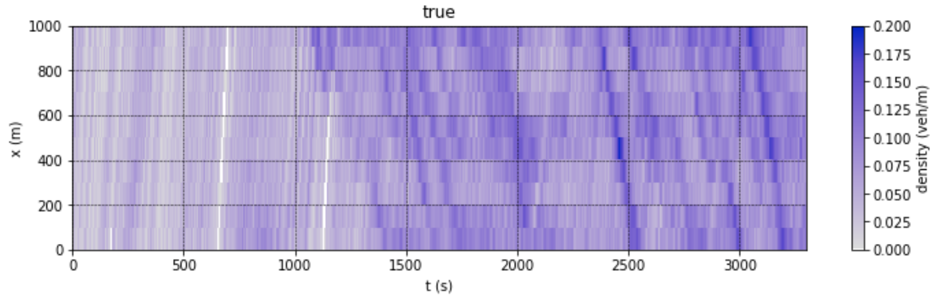
In order to investigate the accuracy of dynamic traffic state reconstruction of the proposed method, time-space diagrams of the estimated densities are shown in Fig. 4. Figure 4b shows the estimated densities when a detector is installed at the upstream boundary of the section, whereas Fig. 4c shows the estimated densities when a detector is installed at the downstream boundary of the section. According to the figures, the both of the estimation results are very similar to the ground truth (Fig. 4a). For example, the both results accurately reconstructed the free-flowing traffic, the congested traffic, and the series of stop-and-go waves.

In order to investigate the effect of the detector location and the RTS smoothing, the relation between the detector location and the overall performance of the proposed method (measured by mean absolute percentage error (MAPE)) is summarized in Fig. 5. According to the figure, the accuracy of the conventional method (without RTS smoothing) became significantly low if a detector was placed at downstream locations. On the other hand, the proposed method (with RTS smoothing) substantially mitigated the accuracy degradation. In fact, the accuracy was not at all decreased if a detector was placed at the middle of the section.

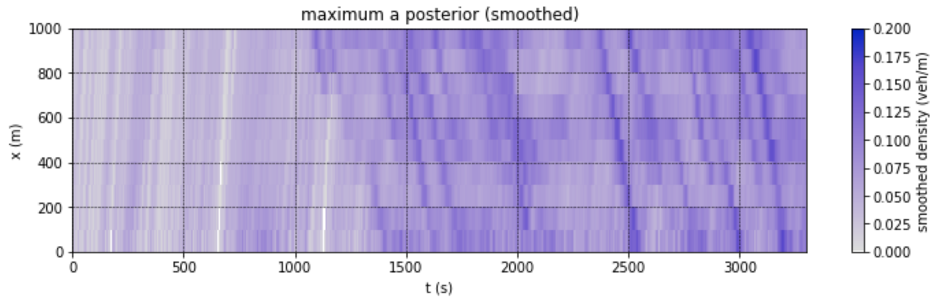
IV. Conclusion

In this study, a traffic state estimation method using connected vehicles and a single detector based on Kalman filtering and RTS smoothing is proposed. The important features of the proposed method are twofold. First, it does not require parameter calibration thanks to the method's data-driven nature. Second, it can estimate accurate traffic state using a single detector, and it does not require detectors in every entries and exits of a road section thanks to RTS smoothing. In addition, the estimation accuracy is not significantly sensitive to detector location. These features were empirically confirmed by actual vehicle trajectories data.

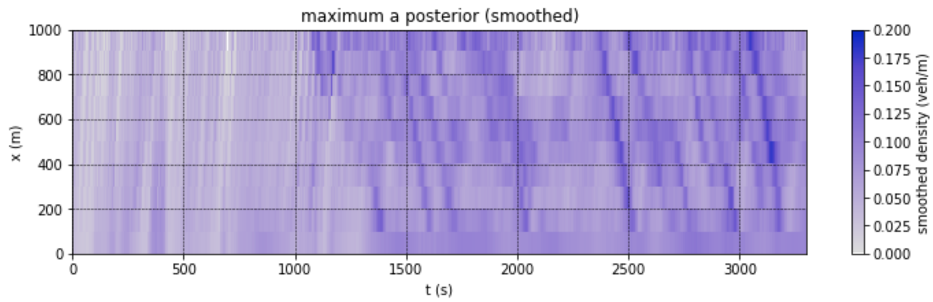
The most important future work is network extension. It will enable estimation of traffic states in links without any



(a) Ground truth



(b) Estimates with upstream detector



(c) Estimates with downstream detector

Fig. 4: Time–space diagrams of density. The right is the future and the top is the downstream.

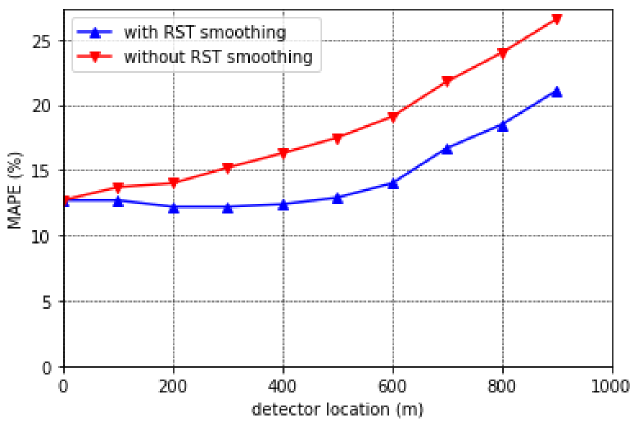


Fig. 5: Detector location and effects of smoothing. $x = 0$ (m) is the upstream end and $x = 1000$ (m) is the downstream end.

detectors. This would be accomplished by estimating route choice ratio of traffic from CV's trajectories as proposed by [19]. Another important problem is mathematical analysis on theoretical observability similar to [7]. Incorporation of automatic fundamental diagram estimation using CV's trajectories [20] would also be considerable to improve the accuracy.

References

[1] Y. Wang and M. Papageorgiou, "Real-time freeway traffic state estimation based on extended Kalman filter: a general approach," *Transportation Research Part B: Methodological*, vol. 39, no. 2, pp. 141–167, 2005.

[2] T. Seo, A. M. Bayen, T. Kusakabe, and Y. Asakura, "Traffic state estimation on highway: A comprehensive survey," *Annual Reviews in Control*, vol. 43, pp. 128–151, 2017.

[3] V. Astarita, R. L. Bertini, S. d'Elia, and G. Guido, "Motorway traffic parameter estimation from mobile phone counts," *European Journal of Operational Research*, vol. 175, no. 3, pp. 1435–1446, 2006.

[4] T. Qiu, X.-Y. Lu, A. Chow, and S. Shladover, "Estimation of freeway traffic density with loop detector and probe vehicle data," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2178, pp. 21–29, 2010.

[5] T. Seo and T. Kusakabe, “Probe vehicle-based traffic state estimation method with spacing information and conservation law,” *Transportation Research Part C: Emerging Technologies*, vol. 59, pp. 391–403, 2015.

[6] N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou, “Highway traffic state estimation with mixed connected and conventional vehicles,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 12, pp. 3484–3497, 2016.

[7] N. Bekiaris-Liberis, C. Roncoli, and M. Papageorgiou, “Highway traffic state estimation per lane in the presence of connected vehicles,” *Transportation Research Part B: Methodological*, vol. 106, pp. 1–28, 2017.

[8] C. Nanthawichit, T. Nakatsuji, and H. Suzuki, “Application of probe-vehicle data for real-time traffic-state estimation and short-term travel-time prediction on a freeway,” *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1855, no. 1, pp. 49–59, 2003.

[9] D. B. Work, O.-P. Tossavainen, S. Blandin, A. M. Bayen, T. Iwuchukwu, and K. Tracton, “An ensemble Kalman filtering approach to highway traffic estimation using GPS enabled mobile devices,” in *IEEE 47th Conference on Decision and Control*, 2008, pp. 5062–5068.

[10] J. C. Herrera and A. M. Bayen, “Incorporation of Lagrangian measurements in freeway traffic state estimation,” *Transportation Research Part B: Methodological*, vol. 44, no. 4, pp. 460–481, 2010.

[11] R. E. Kalman and R. S. Bucy, “New results in linear filtering and prediction theory,” *Journal of Basic Engineering*, vol. 83, no. 3, pp. 95–108, 1961.

[12] H. E. Rauch, F. Tung, and C. T. Striebel, “Maximum likelihood estimates of linear dynamic systems,” *AIAA journal*, vol. 3, no. 8, pp. 1445–1450, 1965.

[13] M. J. Lighthill and G. B. Whitham, “On kinematic waves. II. a theory of traffic flow on long crowded roads,” *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, vol. 229, no. 1178, pp. 317–345, 1955.

[14] P. I. Richards, “Shock waves on the highway,” *Operations Research*, vol. 4, no. 1, pp. 42–51, 1956.

[15] R. J. LeVeque, *Numerical methods for conservation laws*. Springer Basel AG, 1992.

[16] T. Higuchi, Ed., *Introduction to Data Assimilation*. Asakura Publishing, 2011, (in Japanese).

[17] Hanshin Exp. Co. Ltd., “Zen Traffic Data,” 2018, https://zen-traffic-data.net, accessed 2018-09-03.

[18] T. Seo, Y. Tago, N. Shinkai, M. Nakanishi, J. Tanabe, D. Ushirogochi, S. Kanamori, A. Abe, T. Kodama, S. Yoshimura, M. Ishihara, and W. Nakanishi, “Evaluation of large-scale complete vehicle trajectories dataset on two kilometers highway segment for one hour duration: Zen Traffic Data,” in *2020 International Symposium on Transportation Data and Modelling*, 2020, forthcoming.

[19] T. Seo and T. Kusakabe, “Use of small satellites and connected vehicles for large-scale traffic monitoring in road network,” in *IEEE 21st International Conference on Intelligent Transportation Systems*, 2018, pp. 2805–2810.

[20] T. Seo, Y. Kawasaki, T. Kusakabe, and Y. Asakura, “Fundamental diagram estimation by using trajectories of probe vehicles,” *Transportation Research Part B: Methodological*, vol. 122, pp. 40–56, 2019.