Optimal Pricing for Departure Time Choice Problems with Unknown Preference and Demand: Trial-and-error Approach

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ABSTRACT

The trial-and-error approach for congestion pricing finds the optimal toll based only on observable information (e.g., travel time, traffic state) without information on travelers' personal preferences that are often unobservable in practice (e.g., value of time, demand function). This feature makes the method practically useful by solving the well-known issue of "information asymmetry" between the system administrator and the consumers. This paper proposes trial-and-error schemes for the departure time choice problem (also known as the morning commute problem and Vickrey's bottleneck model). We theoretically show that our proposed schemes find the optimal toll in some standard departure time choice problems. Specifically, following cases are considered: fine toll for the homogeneous α–β–γ case, fine toll for the homogeneous case with a non-linear waiting time cost function or a non-linear schedule cost function, and coarse tolling for the homogeneous case with elastic demand. For the first problem, our scheme finds the exact optimal fine toll by one trial. For the second problem, our scheme finds the approximate optimal fine toll by one trial. For the last problem, our scheme converges to the social optimal state fairly quickly.

INTRODUCTION

Congestion pricing is an effective way to alleviate traffic congestion and improve social welfare in transportation systems (*1, 2, 3*). Social optimum can be achieved by congestion pricing if system administrators know travelers' personal attribute and travel preference such as value of time (VoT) and demand functions. However, such attribute and preference are often unobservable to system administrators, and congestion pricing with inaccurate estimates will not achieve the social optimum and may degrade the system efficiency. In economics, this issue is referred to as "information asymmetry" between consumers and administrators (*4*). It makes congestion pricing schemes difficult to implement in practice.

To account for this challenge, several approaches have been proposed in the literature, among which, the *trial-and-error pricing approach* is widely investigated. In this approach, a tolling authority iteratively updates the tolls based on current traffic states. If a proper updating method is applied, the tolls will converge to the optimal tolls, and thus the social optimum will be achieved without knowing personal preference and attribute. Since traffic states are easy to observe or infer (*5*), a trial-and-error pricing scheme would be a practical solution to the information asymmetry problem. Li (*6*) proposed a trial-and-error pricing scheme for static traffic in a link with an unknown demand function, and then Yang et al. (*7*) extended it to general road networks. Additionally, Yang et al. (*8*) incorporated unknown cost functions to this framework so that travelers' VoT is not required to be known. Furthermore, Ye et al. (*9*) developed a trial-and-error pricing scheme for static network traffic considering day-to-day dynamics, instead of assuming route choice is always equilibrated as in (*7, 8*). Other approaches that tackle the information asymmetry issue are the evolutionary game theoretical approach in static network traffic (*10*), self-learning approach for high-occupancy/toll lanes management (*11*), an application of the tradable mobility credit scheme in a bottleneck (*12*), and the tradable bottleneck/network permit schemes in dynamic traffic (*13, 14*).

The *departure time choice problem*, also known as the morning commute problem and Vickrey's bottleneck model, is a well-known transportation problem and has been extensively studied in the literature (*2, 15, 16, 17*). The problem is simple, but remains a valid representation of rush-hour traffic congestion. In a typical departure time choice setting, travelers have to choose their departure time to travel between a single origin and a single destination, which are connected by a single road with a bottleneck. As the travelers desire to arrive the destination in similar time but the road capacity is limited due to the bottleneck, a waiting queue will be formed at the bottleneck if there were no management. If a proper toll is charged, the queue can be eliminated, and thus the social optimal state can be achieved. However, to directly obtain the optimal pricing, precise knowledge on travelers' preference, such as VoT, is required. Therefore, the information asymmetry issue exists in the case of the departure time choice problem.

This study proposes trial-and-error schemes that find the optimal toll in the departure time choice problems under the information asymmetry. The observable information is the queueing pattern, namely, time-varying waiting time. The unobservable information is travelers' personal preference, namely, travel time cost functions, schedule cost functions, desired arrival time, and demand function.

It is noteworthy that Vickrey made following remark, entitled "Trial and Error in Congestion Charge Optimization", in his non-technical monograph published in 1993 (*18*):

> In the case of queues that occur at toll bridges and tunnels, ... these delay times can be multiplied by an estimated average value of delay time per vehicle, and the result used

> as an initial differential toll schedule. Subsequent adjustments can be made by raising the toll at times of day when there is usually a substantial queue, and lowering the toll at times of day when the flow typically falls below capacity.

Our study can be considered as a formalization of this idea in the context of the departure time choice problem and extension to elastic demand cases.

The rest of this paper is organized as follows. First, the problem statements are introduced. Then, we propose a trial-and-error pricing scheme for the simplest case of the departure time choice problem (i.e., homogeneous α–β–γ model), and show that the scheme finds the optimal toll very efficiently. Subsequently, we propose other trial-and-error schemes for several generalized cases of the departure time choice problems (e.g., non-linear cost functions, elastic demand and second-best toll). Finally, achievements and possible future works are summarized in Conclusion section.

THE DEPARTURE TIME CHOICE PROBLEM AND TRIAL-AND-ERROR PRICING SCHEME

The departure time choice problem

The definition of the departure time choice problem in this study is as follows. We use the standard formulation of the problem based on the arrival time to the destination as in, for example, (*19*). The generalized private cost of a traveler is defined as

$$c(t, t^*) = c_w(w(t)) + c_s(t, t^*) + \tau(t), \tag{1}$$

where t denotes the arrival time at the destination, t^* denotes the desired arrival time, $c_w(w)$ denotes the cost function associated with waiting time w, $w(t)$ denotes the waiting time, $c_s(t, t^*)$ denotes the schedule cost function, and $\tau(t)$ denotes the toll. The capacity of a bottleneck is constant and denoted by s. The queuing discipline follows the first-in first-out principle. The number of travelers is denoted by N and may or may not be constant (i.e., demand may be elastic). The travelers are homogeneous, meaning that every travelers have the identical c_w and c_s. Once $\tau(t)$ is given, the traffic quickly reaches the Wardropian equilibrium (*20*). Note that this means that we assume that the day-to-day dynamics of the departure time choice problem is stable, which, however, is being questioned recently in the literature (*21, 22, 23, 24, 25, 26*). However, this issue is out of scope of this study, as it is still an open question.

Trial-and-error pricing scheme

A trial-and-error pricing scheme is loosely defined as a procedure that finds the optimal toll by an iterative procedure that is based on the observable information. In this case, the observable information is the waiting time $w(t)$, the capacity s, and the realized demand N (more precisely, the cumulative arrival and departure curves). The procedure can be loosely described as

Step 1 The administrator charges a time-varying toll $\tau(t)$ based on the current observable information, namely, $w(t)$, s, and N.

Step 2 Some days later, traffic converges into a new departure time choice equilibrium state that reflects the toll. The administrator observes new $w(t)$ and N.

Step 3 The administrator updates the toll $\tau(t)$ based on a pre-determined rule that uses the current observable information $w(t)$, s, and N.

Step 4 Go back to Step 2 until $w(t)$ converges to the social optimal state.

In order to formulate a trial-and-error scheme, it is necessary to design the updating method and show the convergence to the social optimal state, under particular underlying conditions on the departure time choice problem.

THE SIMPLEST CASE

In this section, we show that a trial-and-error scheme finds the optimal toll in the homogeneous α–β–γ model very efficiently.

Specification of the departure time choice problem

We assume that

- The traveler behavior is described by the homogeneous α–β–γ model, in which the cost functions are defined as

$$c_w(t) = \alpha w(t), \tag{2}$$

$$c_s(t, t^*) = \begin{cases} \beta(t^* - t) & \text{if } t < t^*, \\ \gamma(t - t^*) & \text{otherwise}, \end{cases} \tag{3}$$

  where α denotes the VoT, β denotes the early arrival penalty, and γ denotes the late arrival penalty. Conditions $\alpha > \beta > 0$ and $\gamma > 0$ are assumed.

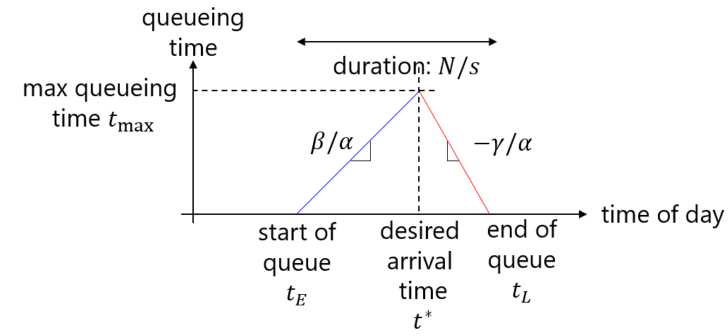
- The desired arrival time of all the travelers is common.

- The number of travelers is denoted by N and is fixed.

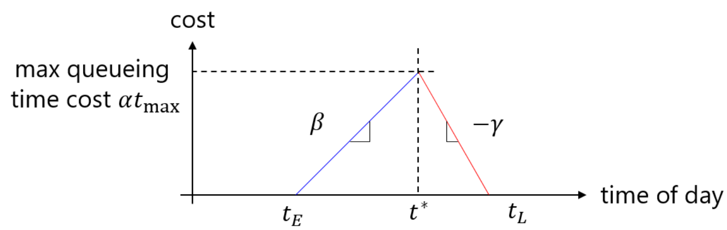
- The road administrator knows that the traveler follows the homogeneous α–β–γ model but do not know the parameter values.

It is commonly known that the equilibrium in this problem can be easily derived by using a simple technique called “isocost curve” (*19, 27*). The isocost curve can be defined as $y(t) = -c_s(t) - \tau(t)$. The waiting time cost at equilibrium can be expressed as $y(t) + c$ where c denotes the generalized travel cost at equilibrium, and the waiting time itself can be expressed as $(y(t) + c)/\alpha$. For the details on the isocost curve method, see, for example, Lindsey (*19*). The no-toll equilibrium queueing pattern in this model is illustrated in Figs. 1a and 1b using time-based and cost-based isocost curves, respectively; note that in this paper we use time-based and cost-based isocost curves depending on the context. Figs. 1a and 1b mean that a triangular queueing pattern is observed in no-toll equilibrium.

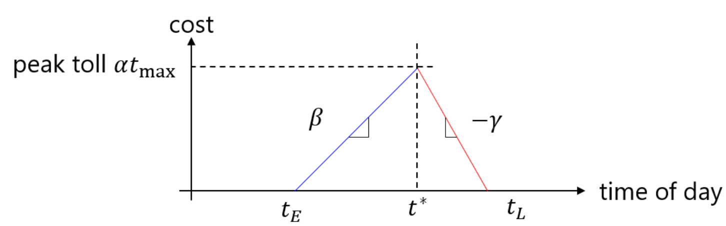
It is also widely known that the social optimal is realized by charging a time-varying congestion toll that is also triangular as shown in Fig. 1c. In the other words, the optimal toll can be identical to the queueing time cost in Fig. 1b. Therefore, the VoT α is required to find the social optimal toll; however, due to the information asymmetry, the VoT is difficult to observe.



(a) Isocost curve (queueing time-based)



(b) Isocost curve (monetary cost-based)



(c) Optimal toll

FIGURE 1 : The homogeneous α–β–γ model.

Trial-and-error pricing scheme

We propose a trial-and-error pricing scheme that finds the optimal toll without external knowledge on VoT α. The scheme charges a particular *trial toll* first, and then find the optimal toll based on the equilibrium pattern under the trial toll.

As shown in Fig. 1, we know that the optimal toll has similar shape with the no-toll queueing time: a triangle whose vertexes are at $(t, \text{cost}) = (t_E, 0)$, $(t_L, 0)$, and $(t^*, \alpha t_{\max})$ where $t_{\max}$ denotes maximum waiting time, and α is unknown to the administrator. By leveraging this knowledge, the optimal toll can be found by the following procedure.

Suppose that the administrator charges a triangular trial toll whose vertexes are at $(t, \text{cost}) = (t_E, 0)$, $(t_L, 0)$, and $(t^*, \tau_{\max})$ where $\tau_{\max}$ is an arbitrary positive value that represents the maximum price of the toll, as illustrated in Fig. 2. Since $\tau_{\max}$ is given arbitrary, this trial toll is not likely to be optimal. It is either under-priced ($\tau_{\max} < \alpha t_{\max}$), over-priced ($\tau_{\max} > \alpha t_{\max}$), or optimal ($\tau_{\max} = \alpha t_{\max}$). Although the administrator does not know which is the case in prior, s/he can eventually know it based on the observed new equilibrium queueing pattern as explained in later.

Assume that the toll is under-priced. The new equilibrium queueing pattern under an under-priced toll can be represented as Fig. 3. The new maximum queueing time is denoted by

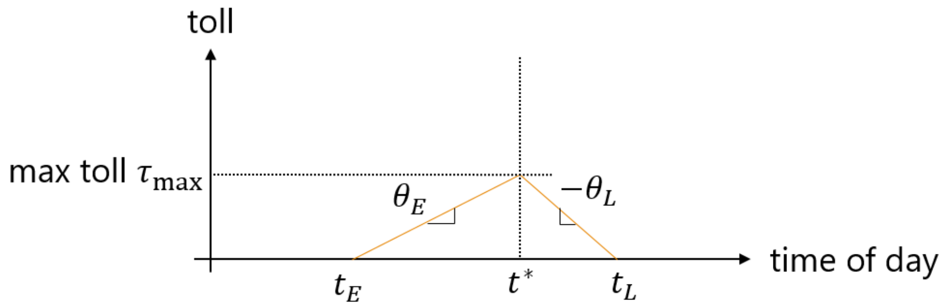


FIGURE 2 : A trial toll in the simplest homogeneous case. $\theta_E = \tau_{\max}/(t^* - t_E)$ and $\theta_L = \tau_{\max}/(t_L - t^*)$.

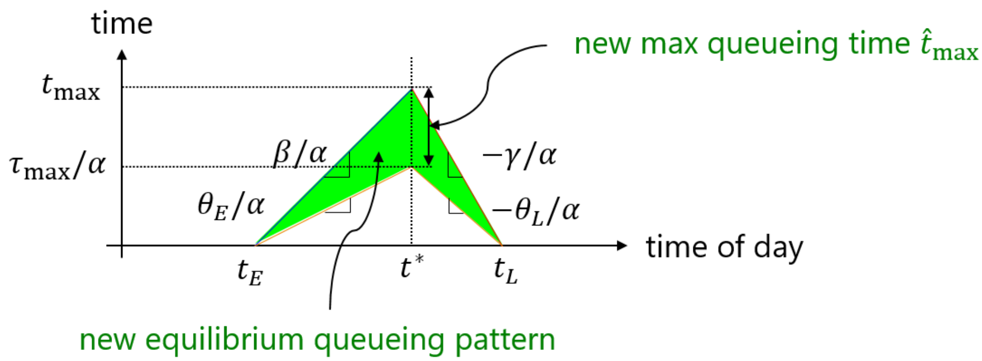


FIGURE 3 : Queueing pattern with an under-priced toll in the simplest homogeneous case.

$\hat{t}_{\max}$. Now, it is obvious that

$$t_{\max} = \frac{\tau_{\max}}{\alpha} + \hat{t}_{\max} \tag{4}$$

holds. Therefore, the value of α can be directly derived as

$$\alpha = \frac{\tau_{\max}}{t_{\max} - \hat{t}_{\max}}. \tag{5}$$

Notice that $t_{\max}$, $\hat{t}_{\max}$, $\tau_{\max}$ are observable. Note that the existence and uniqueness of equilibrium under this trial toll (as well as the other trial tolls discussed in the later of this paper) are guaranteed (*19*).

Contrarily, assume that the toll is over-priced. In this case, a new queueing pattern can be represented as Fig. 4. It can be found that the traffic is not flowing around time t^*, because the toll during the peak period is too expensive. From this observation, the administrator can notice that the trial toll is over-priced. The value of α in this case is derived as

$$\alpha = \frac{t_{\max} - \hat{t}_{\max}}{t_{\max}^2} \tau_{\max} \tag{6}$$

from the observable information.

In summary, the administrator can derive the VoT α regardless of whether the trial toll is under-priced or over-priced (or incidentally optimal). As consequence, the administrator can charge

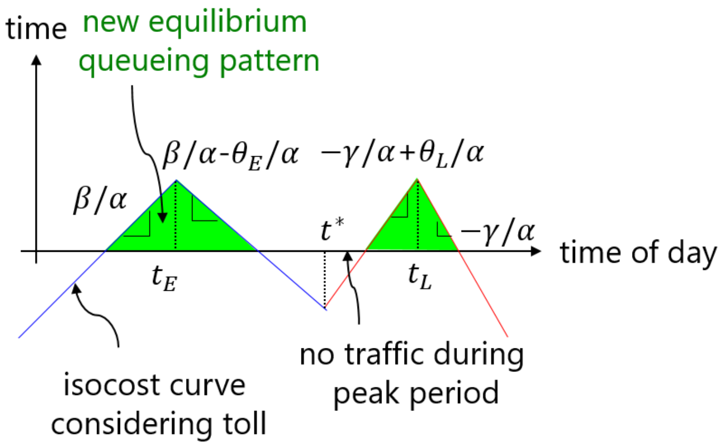


FIGURE 4 : Queueing pattern with an over-priced toll in the simplest homogeneous case.

the optimal toll in the next step. The social optimal is achieved without knowledge on personal preference, namely, VoT, scheduling cost, and desired arrival time. The procedure is summarized as follows:

Step 1 Measure t_E, t^*, t_L from the no-toll equilibrium waiting time. Let $t_{\max}$ be the current maximum queueing time.

Step 2 Charge a trial toll shown in Fig. 2 with arbitrary positive $\tau_{\max}$.

Step 3 Measure the new maximum queueing time $\hat{t}_{\max}$ under the trial toll. Check the following cases.

If there is one queue: The trial toll is under-priced. Derive α by Eq. (5).

If there are two queues: The trial toll is over-priced. Derive α by Eq. (6).

If there is no queue: The trial toll is socially optimal. Derive α by $\tau_{\max}/t_{\max}$.

Step 4 Charge the social optimal toll with $\tau_{\max} := \alpha t_{\max}$.

It is noteworthy that this process requires only *one* trial. Given the definition of the trial-and-error scheme, one trial is the minimum possible number of trials. A trial-and-error scheme with small number of trials is more desirable because, in actual implementation, each trial requires a considerable length of duration in order to get the day-to-day dynamics converged. In this sense, the proposed scheme can be considered as one of the most efficient trial-and-error schemes to find the optimal fine toll in the homogeneous α–β–γ model.

EXTENSIONS

In this section, several extensions of the homogeneous α–β–γ model is considered. Specifically, trial-and-error pricing schemes for cases with distributed desired arrival time, non-linear schedule cost function, non-linear waiting time cost function, and second best pricing for elastic demand, respectively, are proposed.

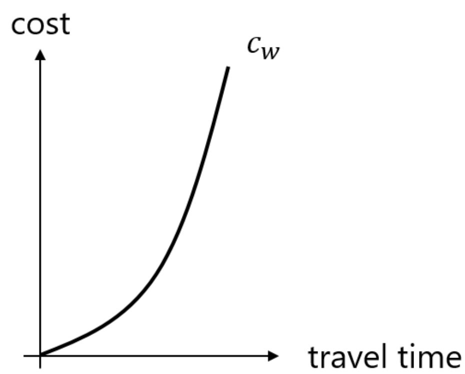


FIGURE 5 : Non-linear waiting time cost function.

Distributed desired arrival time or unknown non-linear schedule cost function
In general, the desired arrival time may be distributed rather than fixed at t^*, or the desired arrival time may be non-linear. However, these features do not change our problem substantially. It is trivial to show that a scheme similar to that in the simplest case (i.e., charge a trial toll whose shape is similar to the no-toll equilibrium queueing time, and then derive VoT by comparing the maximum queueing time before and after the toll) can derive the VoT. The difference is that VoT may not be able to derivable if a trial toll is over-priced; therefore, the administrator need to set a trial toll inexpensive.

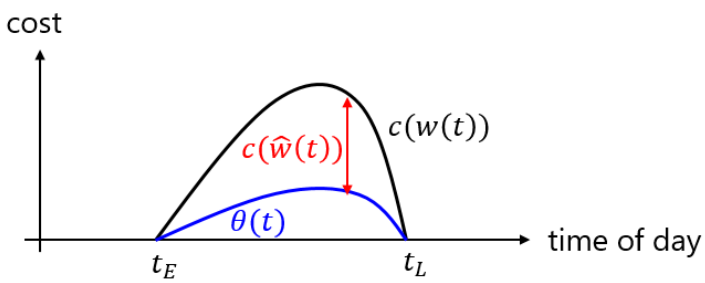
Unknown non-linear waiting time cost function
Non-linear waiting time cost function with unknown functional form is considered. In general, waiting time cost may not be linear due to behavioral or psychological reasons (*28*) (e.g., people may detest too long waiting time, meaning that the waiting time cost function may be convex as shown in Fig. 5), and even its functional form may be unknown. Under this condition, the scheme for the simplest case cannot be applied because the concept of VoT no longer exists. Nevertheless, it is possible to approximate the optimal toll by another trial-and-error scheme as follows.

Let $w(t)$ be the equilibrium waiting time in the no-toll equilibrium. Suppose that the administrator charges an under-priced trial toll, denoted by $\theta(t)$, and new equilibrium waiting time $\hat{w}(t)$ is observed. Note that if a toll is over-priced, the administrator will observe multiple queues as in the simplest case; therefore, the administrator can notice that the toll is over-priced and thus select less expensive toll in the next iteration. In the under-priced situation, the following condition is satisfied because of the feature of the cost-based isocost curves as shown in Fig. 6:

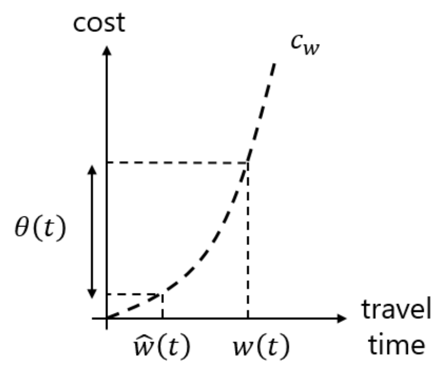
$$c_w(w(t)) - c_w(\hat{w}(t)) = \theta(t), \qquad \forall t \in [t_E, t_L]. \tag{7}$$

The values of $w(t), \hat{w}(t), \theta(t)$ are observable. Note that $c_w(w(t)) > c_w(\hat{w}(t)) > 0$ for $t \in (t_E, t_L)$ and $w(t) > \hat{w}(t)$ for $t \in (t_E, t_L)$ hold because the toll is under-priced.

We can then approximate $c_w(w)$ in $w \in [0, \max w(t)]$ as a piecewise function based on Eq. (7) and observed $w(t), \hat{w}(t), \theta(t)$ as follows. First, note that $c_w(0) = 0$ holds and assume that $c_w(\Delta w) - c_w(0) = \delta \Delta w$ with small Δw and $\delta = \lim_{t \to t_E + 0} \frac{c_w(w(t)) - c_w(\hat{w}(t))}{w(t) - \hat{w}(t)}$ hold. Then, the value of $c_w(w)$ on some discrete w can be sequentially computed by Eq. (7) and the above initial



(a) Equilibrium waiting time cost under a no-toll and an under-priced toll



(b) Observable information regarding waiting time cost function

FIGURE 6 : Estimation of non-linear waiting time cost function.

states. Finally, the function c_w can be estimated by interpolating the computed points. Based on the estimated c_w, the administrators can charge an approximate optimal toll.

Second best pricing for elastic demand

Now we consider a *coarse toll* (also known as step-toll) for the case with *elastic demand* in the α–β–γ model with fixed t^* and a coarse toll. This problem setting is of practical importance (*17, 29*) and thus worths investigating in the context of trial-and-error pricing. A coarse toll is a well-known type of second best toll and is considered as practically easy to be implemented because of its operational simplicity compared with a fine toll. (Note that a fine toll with elastic demand is identical to that with fixed demand; therefore, it is obvious that the trial-and-error scheme discussed in the previous sections finds the optimal toll.)

A coarse toll is defined as

$$\tau(t) = \begin{cases} \tau_H & \text{if } t_{HE} \leq t \leq t_{HL} \\ \tau_L & \text{otherwise,} \end{cases} \tag{8}$$

where τ_H and τ_L represent the toll in the peak hour and the off-peak hour, respectively, and t_{HE} and t_{HL} represent the beginning and the end, respectively, of the peak hour. An example is shown in Fig. 7a along with an equilibrium pattern. An elastic demand is defined as

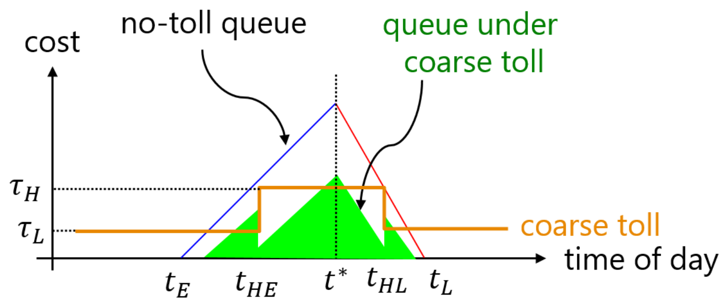
$$N = D(p) \tag{9}$$

where $D(p)$ represents an unknown demand function with generalized travel cost p and is assumed to be monotonically decreasing.

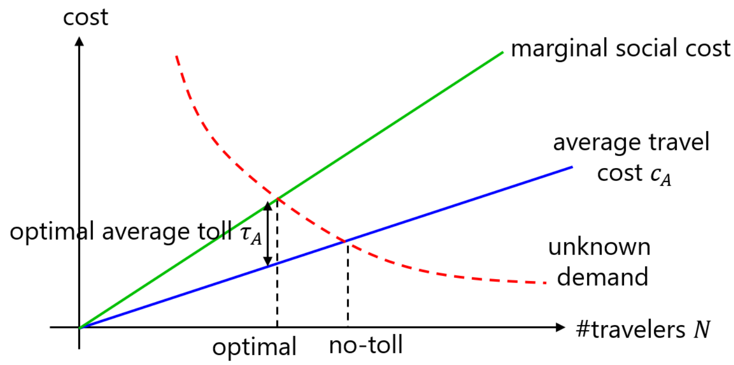
According to Arnott et al. (*17*), a coarse toll under elastic demand is socially optimal if and only if

$$c_A = \tau_A, \tag{10}$$

where c_A represents the average travel cost (sum of the waiting time cost and scheduling cost) among every travelers and τ_A represents the average toll among every travelers. This is a marginal



(a) Isocost curves under coarse toll



(b) Elastic demand, average travel cost, and marginal social cost

FIGURE 7 : Coarse toll with elastic demand.

cost pricing, because the marginal social cost is identical to $2c_A$ (see Fig. 7b). Arnott et al. (*17*) also showed that c_A and τ_A under given demand N can be expressed as

$$c_A = \frac{1}{4}\frac{\beta\gamma}{\beta+\gamma}\left(3 - \frac{(\gamma-\alpha)\beta}{(\beta+\gamma)(\alpha+\beta)}\right)\frac{N}{s}, \tag{11}$$

$$\tau_A = \frac{(t_{HL}-t_{HE})\tau_H + (t_L - t_E - t_{HL} + t_{HE})\tau_L}{t_L - t_E}, \tag{12}$$

and the optimal coarse toll under the optimal coarse toll with given demand N must satisfy following conditions:

$$\tau_H - \tau_L \equiv \rho = \frac{\beta\gamma}{2(\beta+\gamma)}\frac{N}{s}, \tag{13}$$

$$t_E = t^* - \frac{\gamma}{\beta+\gamma}\frac{N}{s} + \frac{(\gamma-\alpha)\rho}{(\beta+\gamma)(\alpha+\gamma)}, \tag{14}$$

$$t_{HE} = t_E + \frac{\rho}{\beta}, \tag{15}$$

$$t_{HL} = t_E + \frac{N}{s} - \frac{2\rho}{\alpha+\gamma}. \tag{16}$$

A trial-and-error pricing scheme needs to find the optimal toll by iteratively updating τ_H, τ_L, t_{HE}, and t_{HL}. In this study, a scheme that consists of two phases is proposed. The first phase is *time preference derivation phase*; it is similar to the trial-and-error scheme for the fixed demand proposed in the previous sections. The second phase is *demand adjustment phase*.

In the time preference derivation phase, the travelers' time preference, namely t^*, α, β, γ, is inferred. The desired arrival time t^* can be easily inferred as the peak waiting time. The other variables are inferred by charging a trial coarse toll similarly to the previous schemes. Let a "step-height" of a trial coarse toll be $\hat{\rho}$, travel time at $t \in [t_{HE}, t_{HL}]$ under the trial toll be $\hat{t}_p$, and travel time of the same t in the no-toll case be t_p. Note that $\hat{\rho}$, $\hat{t}_p$, and t_p are observable. The relation among them is derived as

$$\frac{\hat{\rho}}{\alpha} + \hat{t}_p = t_p. \tag{17}$$

Thus, we get

$$\alpha = \frac{\hat{\rho}}{t_p - \hat{t}_p}. \tag{18}$$

Then, the values of β and γ can be derived from α and the queue evolution speed; specifically, the queue evolution speed $\mathrm{d}w(t)/\mathrm{d}t$ is β/α if $t_E \leq t < t^*$ and is $-\gamma/\alpha$ if $t^* < t \leq t_L$ as illustrated in Fig. 1a.

The additional challenge compared to the previous schemes is that the demand is elastic with an unknown functional form. This is solved by the demand adjustment phase as follows. As shown in Fig. 7b, the demand function is monotonically decreasing, the marginal cost is always as twice as the average travel cost, and thus the social optimal is achieved if Eq. (10) is satisfied. Suppose that the administrator charges a trial toll with arbitrary τ_L with $\tau_H = 0$ (i.e., uniform toll). Since α, β, γ are known by the previous phase, it is possible to compute c_A and τ_A under the current N and τ_L by Eqs. (11)–(16), and thus it is possible to determine whether the current toll is over-priced ($c_A < \tau_A$) or under-priced ($c_A > \tau_A$). Therefore, because of the monotonicity of the demand and the marginal cost functions, the optimal N can be found by the bisection method (i.e., a simple numerical method to solve an equation) that iteratively updates τ_L. Specifically, let τ_L^{UP} be the trial toll of the most recent iteration that is under-priced, and τ_L^{OP} be the trial toll of the most recent iteration that is under-priced. The trial toll in the next iteration is determined as $\tau_L^{\mathrm{new}} = (\tau_L^{\mathrm{UP}} + \tau_L^{\mathrm{OP}})/2$. Then, substitute τ_L^{UP} or τ_L^{OP} with τ_L^{new} depending on whether τ_L^{new} is under-priced or over-priced, and repeat the procedure. Because of the monotonicity, this procedure is guaranteed to converge to the optimal toll. This bisection method can be directly used as a trial-and-error scheme as in (*6, 7*), because it does not require the knowledge on the demand function except for the monotonicity.

The procedure of the proposed trial-and-error scheme can be summarized as follows:

Step 1 [Time preference derivation phase] Charge a trial coarse toll. The value of α and t^* can be determined from the new equilibrium pattern (Eq. (18)). Subsequently, the values of β and γ can be determined. Set $\tau_L = 0$.

Step 2 [Demand adjustment phase]

Step 2.1 Charge a uniform toll (i.e., $\tau_L = \tau_H \geq 0$) with current τ_L. Measure the realized demand N.

Step 2.2 From the known information, a virtual average travel cost under the optimal coarse toll, denoted as $\hat{c}_A$, and the mean of the optimal coarse toll, denoted as $\hat{\tau}_A$, in the current demand can be computed (Eqs. (11)–(16)).

Step 2.3 Check $\hat{c}_A \simeq \hat{\tau}_A$ with some convergence criteria. If it is true, go to Step 3. If it is false, use the bisection method to update τ_L and go back to Step 2.1 (essentially, increase τ_L if $\hat{c}_A > \hat{\tau}_A$, or decrease τ_L otherwise).

Step 3 Charge an approximate optimal coarse toll by determining the optimal τ_H, t_{HE}, and t_{HL} by the current τ_L and Eqs. (13)–(16).

A result of a numerical experiment of the proposed scheme is shown in Fig. 8. According to Fig. 8a, we can confirm that the toll quickly converges to the social optimal state (i.e., $\hat{\tau}_A = \hat{c}_A$). It means that even if the administrator terminates the trial-and-error process with a mild convergence criteria, the social welfare will be substantially improved. Figure 8b shows the isocost curve in the converged state; it approximately satisfies the social optimality.

CONCLUSION

This study proposes trial-and-error schemes for optimal pricing in the departure time choice problems. The advantage of trial-and-error schemes is that it does not require precise information on travelers' personal preferences (i.e., waiting time cost function, schedule time cost function, demand function), which are difficult to observe in practice. Some particular departure time choice problems are considered: the homogeneous α–β–γ model, cases with non-linear, unknown waiting time cost function and schedule cost function, and second best tolling in elastic demand cases. We theoretically show that the proposed schemes quickly find the optimal toll in these cases. Especially, only one trial is required to find the fine toll in the homogeneous α–β–γ model; this means that the proposed scheme is one of the most efficient trial-and-error schemes for this problem.

Several further extensions are worth considering. First, extension to heterogeneous commuter cases (*27*) is valuable. This is because heterogeneity is an important issue in the departure time choice problem although it is unobservable to road administrators. Second, explicit consideration of day-to-day dynamics (instead of assuming that the day-to-day dynamics always converges) is important as Ye et al. (*9*) did in the static network traffic case. For this purpose, the reinforcement learning would be useful, since it is an efficient way to implement trial-and-error procedures. From a theoretical point of view, this could be a challenging task that involves the stability of the dynamics, which is receiving attention in the recent literature (*21, 22, 23, 24, 25, 26*); from an application point of view, it enables us to develop a fast-converging pricing scheme in real-world implementation. This study can provide a stepping stone to these future extensions.

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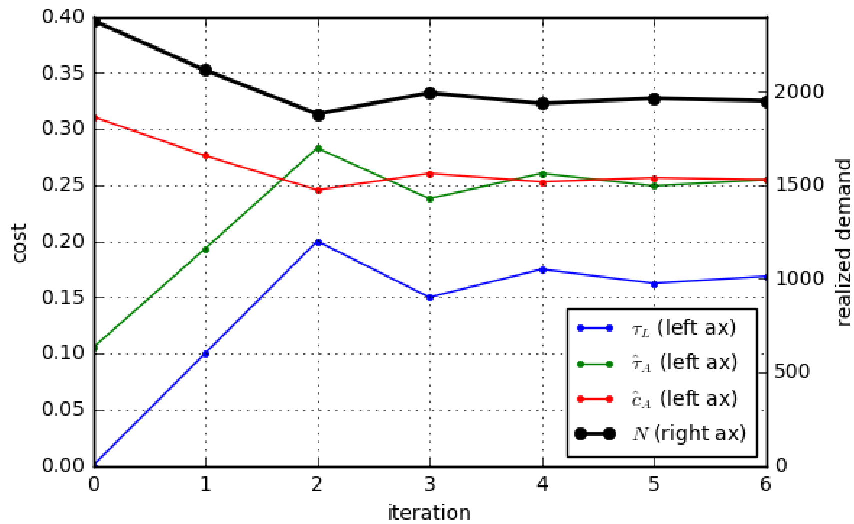
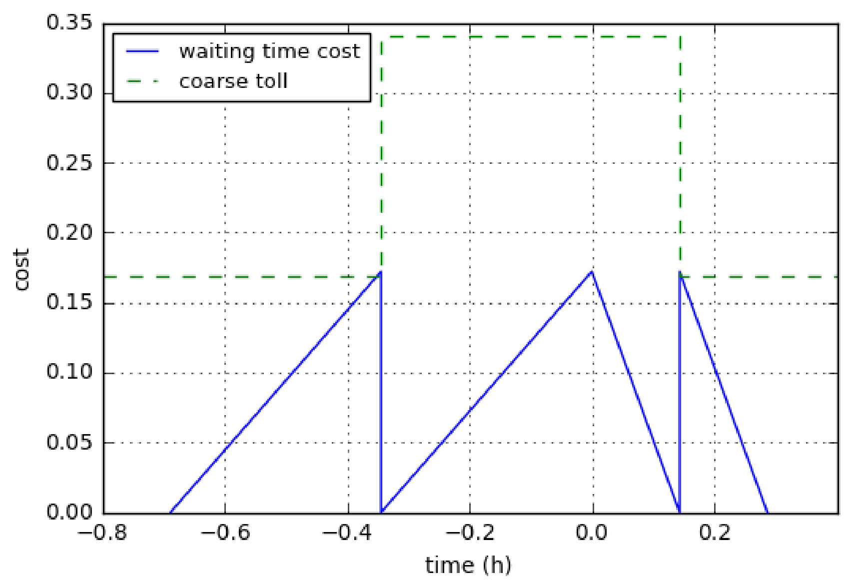
AUTHOR CONTRIBUTION STATEMENT

The authors confirm contribution to the paper as follows: study conception and design: T. Seo, Y. Yin; analysis and interpretation of results: T. Seo, Y. Yin; draft manuscript preparation: T. Seo.



(a) Trial-and-error process



(b) Isocost curve and toll in the converged state

FIGURE 8 : Numerical example of the coarse tolling with the elastic demand. The model specification: $D(p) = \eta/p$ with $\eta = 1000$ (person/cost), $s = 2000$ (veh/h), $\alpha = 1$ (cost/h), $t^* = 0$, $\beta = 0.5$ (cost/h), $\gamma = 1.2$ (cost/h).

All authors reviewed the results and approved the final version of the manuscript.

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