Flow Breakdown

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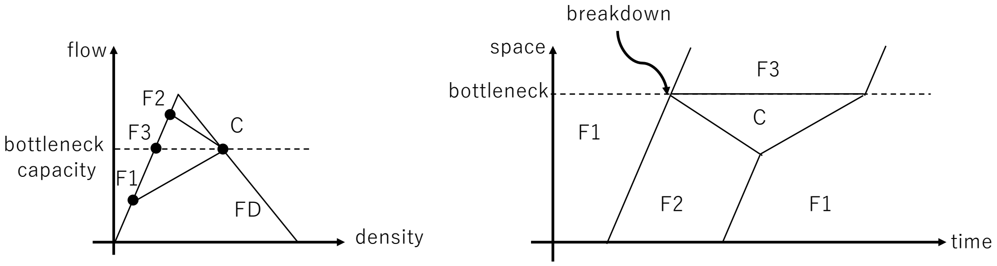
Abstract

Flow breakdown is defined as a transition from a free-flow state to a congested state of a traffic state at a certain location. It occurs when the arrival flow to the location exceeds its capacity. This article contains discussions on characteristics of flow breakdown, such as main causes, stochastic nature, and long-term variations. We also discuss peculiar traffic flow phenomena, associated with the flow breakdown: “stop-and-go traffic” and “capacity drop." Finally, future research directions regarding the flow breakdown, which is based on emerging data and vehicle technologies are discussed.

Keywords
Flow breakdown, Traffic congestion, Bottleneck, Traffic disturbance, Stochastic capacity, Stop-and-go traffic, Capacity drop

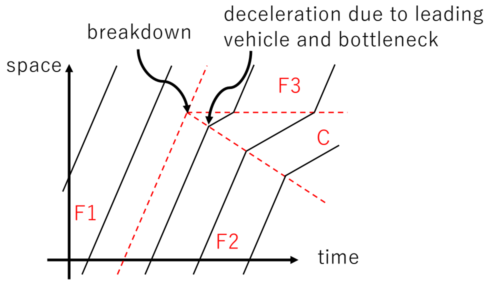
Chapter for *International Encyclopedia of Transportation*, Vickerman, R., Noland, R.B., Ettema, D. (Ed.), Elsevier Science.

August 14, 2020



(a) Flow-density diagram

(b) Time-space diagram



(c) Vehicle trajectories near breakdown point

Fig.1 Breakdown in time–space and flow–density diagrams

1. Introduction

Flow breakdown is defined as a spontaneous transition from a free-flow state to a congested state of a traffic state at a certain location. “Spontaneous” in this definition means that the transition is not caused by a queue extension from the downstream sections. This occurs when the arrival flow to the location exceeds the capacity of the location. Therefore, a breakdown happens at a bottleneck (see the article “Bottleneck”). In the Highway Capacity Manual (HCM) (Transportation Research Board, 2016), breakdown is explained as “*the transition from uncongested to congested conditions. The formation of queues upstream of the bottleneck and the reduced prevailing speeds make the breakdown evident.*” and “*breakdown occurs when the ratio of existing demand to actual capacity … exceeds 1.00 .*”

The definitions of free-flowing and congested states of traffic are as follows. Traffic is in a free-flow state when most of the vehicles are traveling at their own desired travel speed. On the other hand, traffic is in congested state when most of the vehicles are forced to travel at speeds lower than their desired travel speed, because of too small spacing (i.e., the distance headway is too small). Macroscopic traffic states can be divided into two qualitative categories: free-flowing and congested states. In general, traffic with a small density is free-flowing and traffic with a large density is congested. The threshold density between the free-flowing and congested states is called critical density.

Breakdown is a spatial-temporal phenomenon. Fig. 1 illustrates breakdown at a bottleneck, based on kinematic wave theory (for a basic interpretation of this figure, see the article “Bottleneck”). In the flow–density (or fundamental) diagram (Fig. 1a), breakdown can be represented as the transition from the free-flowing states (F1 and F2) to the congested state (C). However, this representation is not complete, because it is not clear where and when the transition happens. In the time–space diagram (Fig. 1b), breakdown can be represented as the onset of congestion at the bottleneck location. If a traffic detector was placed at the direct upstream position of the bottleneck,

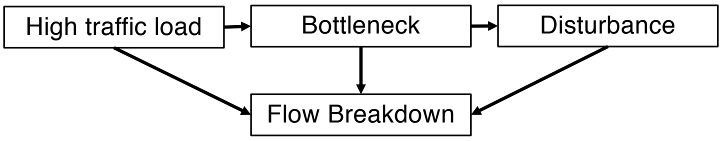


Fig.2 Schematic representation of relationship between three factors and flow breakdown

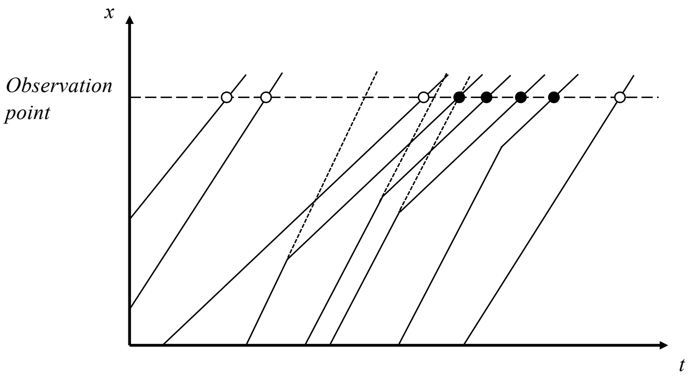


Fig.3 Trajectories of traffic flow. Solid lines indicate real vehicle trajectories, dotted lines indicate the trajectories of vehicles driving at their desired speed, open circles indicate freely driving vehicles in the study section, and solid circles indicate following vehicles.

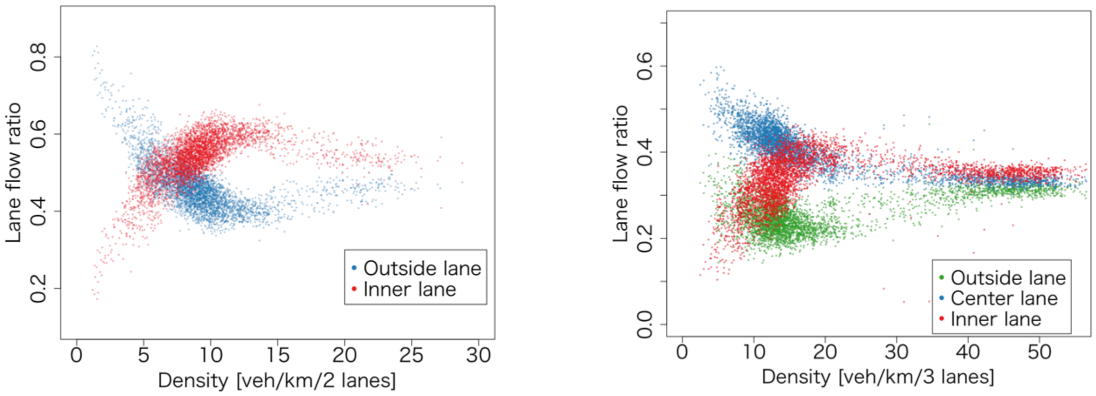
it would observe the aforementioned transition of traffic state from free-flowing to congested. Note that this breakdown is not caused by a queue extension from the downstream sections; the downstream section is always in the free-flowing state. On a microscopic scale (Fig. 1c), flow breakdown can be roughly understood as the sudden speed reduction of a particular vehicle for which its following vehicles must also reduce their speed, although its leading vehicle is free-flowing. This speed reduction grows into a macroscopic waiting queue.

2. Characteristics of flow breakdown

Traffic congestion occurs when traffic demand exceeds traffic capacity. This seems to be an unquestionable fact and, pragmatically, it is true. However, observations of real-world traffic reveal that traffic flow is not as simple, due to its stochastic nature. Traffic demand varies stochastically, and even traffic capacity does. That is why flow breakdown does not always happen at the same traffic demand level even at a same bottleneck point.

2.1 Necessary conditions for flow breakdown

According to Treiber and Kesting (2013), traffic flow breakdowns are caused by "the simultaneous action of three factors: high traffic load, a bottleneck, and disturbances of traffic flow caused by individual drivers." Fig. 2 illustrates this interrelationship. When traffic flow with a high density encounters a bottleneck, some speed disturbance may occur for a variety of reasons. If the high traffic volume still continuously flows into the bottleneck, the disturbance remains there and is amplified. Then, a severe deceleration wave is generated and propagates upstream. Finally, traffic flow breaks down and the discharge flow rate from the bottleneck decreases (see Section 3.1 for details).



(a) Example of two-lane section

(b) Example of three-lane section

Fig. 4 Observation of lane-flow distribution on freeways (Data from Shiomi et al., 2019)

2.1.1 High traffic load

Vehicles arrive at a bottleneck section randomly, not uniformly. This randomness is caused by the heterogeneity of time headways and desired speeds. A vehicle with a higher desired speed catches up with a slower vehicle and is forced to slow down and follow it. The number of following vehicles increases behind the slow vehicle, and then a platoon in which traffic load is locally and temporarily high is generated (Shiomi et al., 2011), as shown in Fig. 3. In a large platoon, speed disturbances at a bottleneck are likely to occur, propagate, and be amplified. This causes traffic flow breakdown.

On a multilane highway, traffic load is unevenly distributed lane by lane (Shiomi et al., 2015). When traffic density approaches critical density, more traffic is loaded onto the inner lane than the outer and center lanes, as shown in Fig. 4. Inequality in lane use results, at first, in a breakdown of traffic flow in the inner lane. Then, some of the overflow quickly moves to the less congested lane, thereby causing all the lanes to become congested almost simultaneously (Xing et al., 2014).

2.1.2 Bottlenecks and local disturbances

Local disturbances are likely to occur at a bottleneck or, conversely, we can say a bottleneck is the place where local disturbances frequently occur. On freeways, the cause of the disturbance at a bottleneck depends on the type of bottleneck. At a lane drop point, work zone, or accident site, where traffic capacity is apparently lower than the section behind the bottleneck, drivers are forced to change lanes and cut in at the bottleneck, which generate speed disturbances. At diverging and weaving sections, lane changes are necessary to get to the destination, which causes speed disturbances as well. At sags, tunnels, and even “rubbernecking” sites, drivers may unconsciously drop their driving speed, in turn stimulating any surrounding drivers to change lanes. Both speed drops and lane changes may cause considerable speed disturbance (Patire and Cassidy, 2011). For more details on a bottleneck refer to the article “Bottleneck.”

2.2 Stochastic nature of flow breakdown

The breakdown flow rate, which is the flow rate at which traffic flow falls into breakdown and a congestion queue appears, varies even under the same road and environmental conditions, because each necessary condition mentioned above is not deterministic, but stochastic, and vehicle behaviors

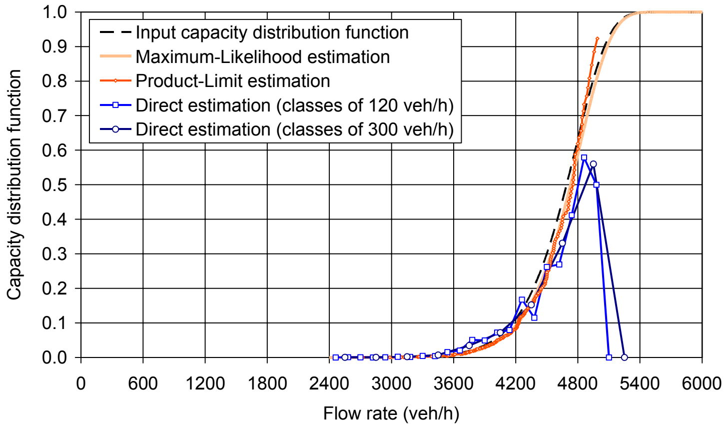


Fig. 5 Input and output capacity distribution functions for simulation model (Adapted from Geistefeldt and Brilon, 2009)

including freely driving, car-following, and lane changing are heterogenous. Thus, flow breakdowns are probabilistic events, and the concept of stochastic capacity has been proposed (Brilon et al., 2005). In this concept of stochastic capacity, the capacity distribution function $F_c(q)$ is defined and represented as follows:

$$F_c(q) = p(c \leq q), \tag{1}$$

where $p(\cdot)$ is the probability, c is capacity, and q is flow rate. In accordance with the definition of "deterministic" traffic capacity, that every flow rate greater than the capacity causes a traffic breakdown, the capacity distribution function $F_c(q)$ represents the probability of a traffic breakdown dependent on the flow rate q.

According the HCM (Transportation Research Boards, 2016), the breakdown probability is estimated by allocating the observed volumes into groups, determining the ratio of the number of breakdown intervals and the total number of intervals for each group, and fitting that to the Weibull distribution. Herein, the fifteenth-percentile value of the breakdown probability is recommended to be the traffic capacity.

However, observed traffic capacity is considered as a type of censored observation. That is, when a traffic flow rate is observed without breakdown, it implies that the traffic capacity at that time is greater than the observed flow rate. This "direct" estimation is sometimes criticized because it ignores bias. To remove bias, a method to estimate breakdown probability has been developed, based on models for lifetime data analysis (Brilon et al., 2005). A non-parametric method to estimate the distribution function of lifetime variables is the so-called "Product Limit Method" (PLM), in which the capacity distribution function $F_c(q)$ is written as Eq. (2).

$$F_c(q) = 1 - S(q) = 1 - \prod_{i:q_i \leq q} \frac{k_i - d_i}{k_i}, i \in \{B\}, \tag{2}$$

where q is flow rate [veh/h], q_i is flow rate in interval i (veh/h), k_i is the number of the intervals with a flow rate of $q \geq q_i$, d_i is the number of breakdowns at a flow rate of q_i, and $\{B\}$ is a set of breakdown intervals. For a parametric estimation, the distribution parameters are estimated by applying the Maximum Likelihood Estimation (MLE). Eq. (3) defines the likelihood function L:

$$L = \prod_{i=1}^{n} f_c(q_i|\boldsymbol{\beta})^{\delta_i} \cdot [1 - F_c(q_i|\boldsymbol{\beta})]^{1-\delta_i}, \tag{3}$$

where $\boldsymbol{\beta}$ is a parameter vector of the distribution function of the capacity; n is the number of intervals; and $\delta_i = 1$, if interval i contains an uncensored value; and, $\delta_i = 0$, if interval i contains a censored

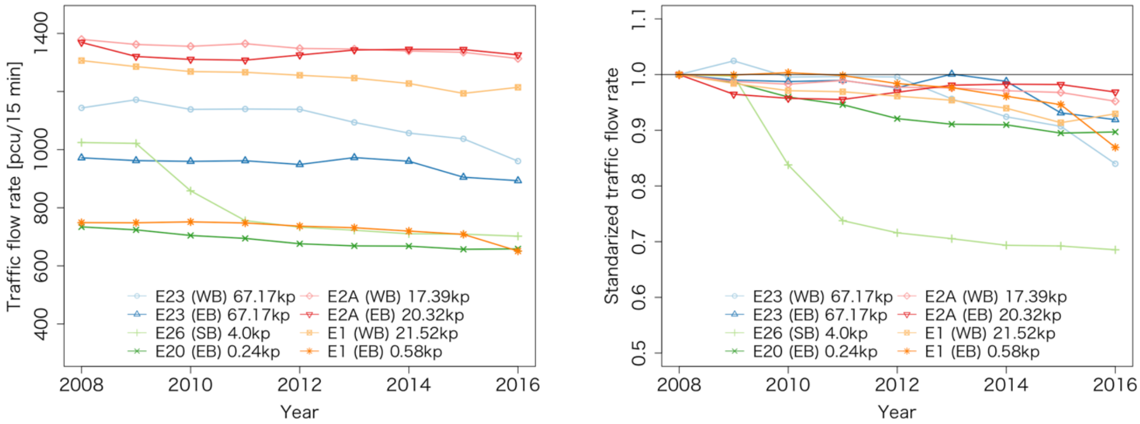


Fig. 6 Long-term variation of fifth-percentile value of breakdown probability (Data from Shiomi et al., 2019)

value. Comparisons between the direct estimation, PLM, and MLE are illustrated in Fig. 5 (Geistefeldt and Brilon, 2009), which shows that the PLM and MLE gives more robust results than direct estimation.

2.3 Long-term variations of breakdown flow rate

Recently, it was reported that the breakdown flow rate has been gradually decreasing, over time, in some countries. Fig. 6 shows the long-term variation of the fifth-percentile value of breakdown probability, from Jan. 2008 to Dec. 2016, at nine typical bottlenecks on freeways in Japan (Shiomi et al., 2019). Fig. 6 indicates that the fifth-percentile traffic volume of breakdown probability decreased over time at eight of the nine sites. On average, the fifth percentile of breakdown probability decreased by 10.8 veh/15 min each year (i.e., 86.4 veh/15 min over eight years), which is equivalent to 91.7% of the initial value. The environment at these bottlenecks has remained unchanged in the eight years, so the decreasing trend may be attributed to the characteristics of vehicles, drivers, or both. This suggests that local disturbances are more likely to occur today than in the past, because the characteristics of traffic flow have changed.

3. Associated phenomena and their possible explanations

Peculiar traffic flow phenomena, associated with a traffic breakdown, are usually observed near an active bottleneck, which has negative effects on traffic efficiency and safety. Here, we will discuss two peculiar phenomena: “capacity drop” and “stop-and-go traffic,” and their possible explanations.

3.1 Capacity drop

Once traffic breaks down at a bottleneck, one would logically expect that the queue discharge flow rate from the bottleneck is equal to its traffic capacity. However, in real traffic, the observed queue discharge flow rate decreases by an order of 10%. This phenomenon is called “capacity drop.” This phenomenon has been recognized for many years (e.g., Lincoln and Holland Tunnels, Edie and Foote, 1958), and the shape of associated flow-density relation typically resembles a mirror image of the lambda character (λ), which exhibits a discontinuity between the free-flow and congested states (see, for example, Koshi et al., 1983). The decrease in the queue discharge flow rate prolongs the congestion period, and can incur a profound additional delay of drivers, which has motivated development of several traffic control strategies, such as ramp-metering and variable speed limits.

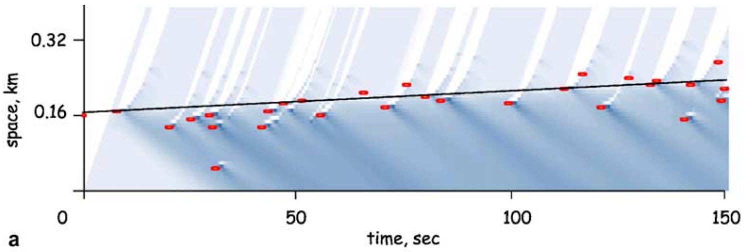
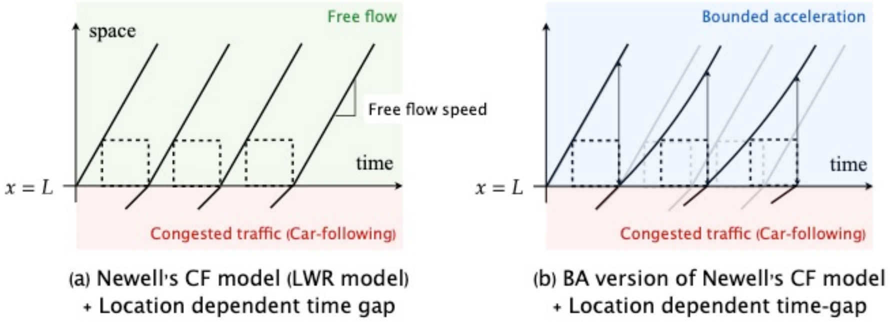


Fig.7 Lane-changing particles (red dots) and voids (white regions) in traffic stream (Adapted from Laval and Daganzo, 2006)



(a) Newell's CF model (LWR model) + Location dependent time gap

(b) BA version of Newell's CF model + Location dependent time-gap

Fig. 8 Illustration of the formation of the capacity drop

The hypotheses for the occurrence of the capacity drop phenomenon are mainly divided into two categories, depending on assumed driving behaviors, i.e., a (longitudinal) sluggish car-following behavior, and a (lateral) disruptive lane-changing behavior. Let us look at the latter category first. In Laval and Daganzo (2006), it is postulated that lane-changing vehicles create voids in (congested) traffic streams, and that these voids reduce flow. They describe this mechanism with a hybrid model in which lane-changing particles, endowed with a bounded acceleration (BA) capability, are endogenously generated and incorporated into the kinematic wave model as moving bottlenecks. Additionally, they found that the model can reproduce the capacity drop phenomenon (see Fig. 7), in agreement with empirical observations at a merge bottleneck (Cassidy and Rudjanakanoknad, 2005). However, in Cassidy and Rudjanakanoknad (2005), it was also argued that "lane changing alone might not explain the capacity drop." In fact, a significant capacity drop has been observed even in a single-lane freeway bottleneck (Okamura et al., 2000).

Regarding car-following behavior, it is commonly conjectured that a reduction in the queue discharge flow rate is caused by a low acceleration and/or delayed response of vehicles that leave from queues or traffic disturbances (e.g., oscillations), upstream of bottlenecks (Hall and Agyemang-Duah, 1991; Koshi et al., 1992). One modelling approach assumes state-dependent changes due to different car-following styles. For instance, Zhang and Kim (2005) describes less sensitive responses to the leading vehicle in car-following models, by a state-dependent time-gap parameter, that depend on the distance-gap (and speed). Chen et al. (2014) also consider some drivers, who become less aggressive and adopt longer response times and minimum spacings when passing traffic disturbances, in their multi-class traffic flow model. However, none of these models treat the capacity drop phenomenon endogenously. Furthermore, because their mechanisms are purely based on (intra- and inter-) drivers' heterogeneities, insights to the relation between the capacity drop and bottlenecks — spatial heterogeneities— are lacking.

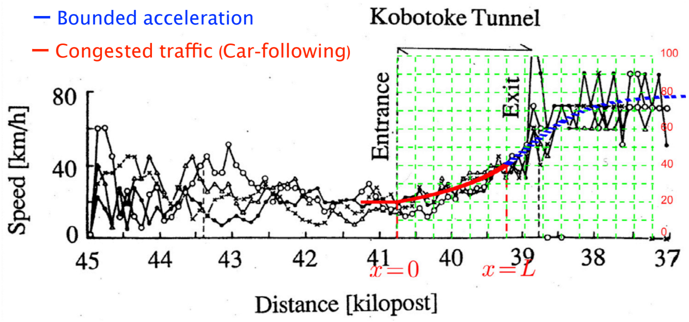


Fig. 9 Comparison between empirical and model's speed profiles in capacity drop stationary state near bottleneck (Image adapted from Jin, 2018)

Another modelling approach, which can endogenously describe the capacity drop phenomenon, is founded on two main assumptions: spatial heterogeneities and the BA of vehicles (Jin, 2017, 2018). The spatial heterogeneity is expressed by a location-dependent fundamental diagram. For example, the jam density diminishes within lane-drop bottlenecks. Additionally, the time gap may increase within sag (and tunnel) bottlenecks, because drivers cannot fully compensate for gradient changes, but try to keep their spacing. Such a location-dependent fundamental diagram describes not only the local reduction in the (static) capacity (i.e., bottleneck effects), but also the delay in the speed adaption to the leading vehicle in the car-following situation (Wada et al., 2020). As illustrated in a lead vehicle problem by Fig. 8(b), the BA version of Newell (2002)'s car-following model (Laval and Leclercq, 2008), with a specific location-dependent fundamental diagram, leads to a capacity drop. Specifically, if the demand exceeds the capacity, (i) the speed of a vehicle arriving at the end of the bottleneck (at x = L) is less than that of the leading vehicle, due to delayed speed adaption; and, (ii) unlike in Fig. 8(a), the vehicle cannot recover to the free-flow speed instantaneously, owing to the BA constraint, which results in an increase in the headway of the next vehicle at x = L (i.e., the queue discharge flow rate decreases). This vicious cycle (steps (i) and (ii)) continues to reach the capacity drop stationary state. Furthermore, this modelling approach can explain a very low acceleration rate, when passing through the bottleneck, during the capacity drop stationary state (Persaud and Hurdle, 1988; Koshi et al., 1992). Fig. 9 demonstrates that the speed profile in the capacity drop stationary state, by the (calibrated) model (Jin, 2018; Wada et al., 2020), very well matches that observed in the Kobotoke tunnel in Japan (Koshi et al., 1992).

3.2 Stop-and-go traffic

Once traffic flow breaks down, deceleration waves are formed around an active bottleneck and propagate regularly in the upstream direction, with an almost constant speed (on the order of 20 km/h). Vehicles are forced to decelerate and accelerate when passing through these traffic disturbances. This phenomenon is called "stop- and-go traffic" or "traffic oscillation," and was first found in observations at the Lincoln tunnel (Edie and Foote, 1958). It is also known that the oscillations are rather small in amplitude immediately upstream of the bottleneck, but they are amplified as they propagate against the traffic stream; and, the oscillations across lanes are synchronized. Furthermore, Tian et al. (2016) recently found from empirical and experimental data that the standard deviation of the vehicles' speeds increases in a concave manner, during the oscillations (see Fig. 10).While the trigger of stop-and-go traffic can be caused by any traffic disturbance, such as lane changes and/or slow vehicles, there is still active discussion regarding the mechanism of the propagation and amplification of these disturbances.

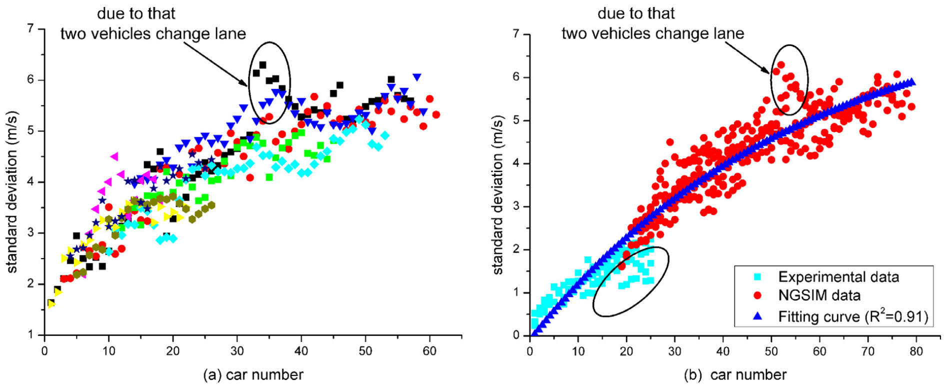


Fig. 10 The standard deviation of the speed of each vehicle (Adapted from Tian et al., 2016)

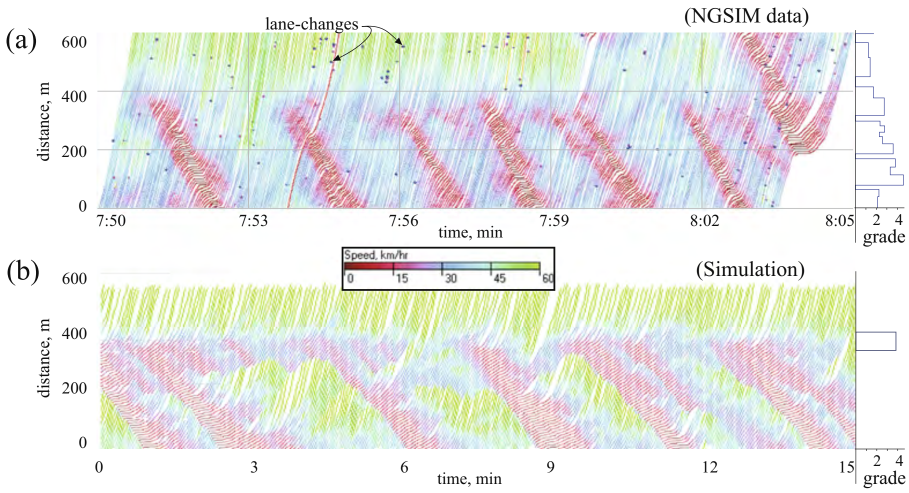


Fig. 11 Vehicle trajectories of NGSIM data and simulation results (Adapted from Laval et al., 2014)

Conventionally, stop-and-go traffic has been studied as a phase transition or pattern formation, due to the system instability (i.e., the string or flow instability) of car-following models, that are described as a system of (ordinary or delayed) differential equations (Kometani and Sasaki, 1958; Bando et al., 1995; Treiber and Kesting, 2013). This line of research indicates that, regardless of the details of the models, some common driving behaviors such as finite acceleration, deceleration, and reaction times lead to stop-and-go traffic (Treiber and Kesting, 2013). However, an empirical validation of these models is lacking (Laval and Leclercq, 2010). Furthermore, the growth pattern of oscillations, in the typical (homogeneous and deterministic) car-following models, is qualitatively different from the findings above (i.e., a convex growth pattern in the initial stage) (Tian et al., 2016).

A more behavioral modelling approach is based on extensive empirical analyses of vehicle trajectory data that is available more recently (e.g., NGSIM data, U.S. Department of Transportation Federal Highway Administration, 2016). Specifically, it has been proposed that oscillations may be caused by changes in the car-following behaviors of drivers when facing traffic waves. Yeo and Skabardonis (2009) proposed an asymmetric traffic theory in which acceleration and deceleration behaviors have different characteristics due to the anticipation and overreaction of drivers. In contrast to the introduction of the intra-driver heterogeneity, Laval and Leclercq (2010) proposed a car-following model with “timid” and “aggressive” drivers, as observed in empirical trajectory data. This study showed that combining a model, without an instability mechanism (i.e., the BA version of

Newell's car-following model, Laval and Leclercq, 2008), with such an inter-driver heterogeneity model can produce realistic oscillations.

The last but simplest modelling approach is to introduce human errors as random noise. The earliest model by this approach is by Nagel and Schreckenberg (1992), which reproduces stop-and-go traffic by a cellular automata model, with a breaking probability. More recently, Laval et al. (2014) simply added an acceleration noise, which is a function of road geometry, to the BA version of Newell's car-following model, and found that the oscillations produced accord well with observation (see Fig. 11). Furthermore, Tian et al. (2016) showed that this model can reproduce the concavity of the disturbance growth well.

4. Future direction

In the future, two innovations may occur regarding flow breakdown. First, our understanding of breakdown may be improved by using new data. Second, we may be able to control breakdown by using connected and automated vehicles (CAVs).

Conventionally, traffic data were mainly collected by traffic detectors that are fixed at a location, and their pulse data or aggregated data were used for analysis. Although they enabled us to understand basic properties of breakdown as reviewed in the previous sections, they have clear limitations. That is, they do not capture the continuous spatial-temporal dynamics of traffic. Because breakdown is a continuous spatial-temporal phenomenon that covers approximately one hundred meters and a minute, it is impossible to reveal the detailed, full picture of a breakdown by conventional detectors.

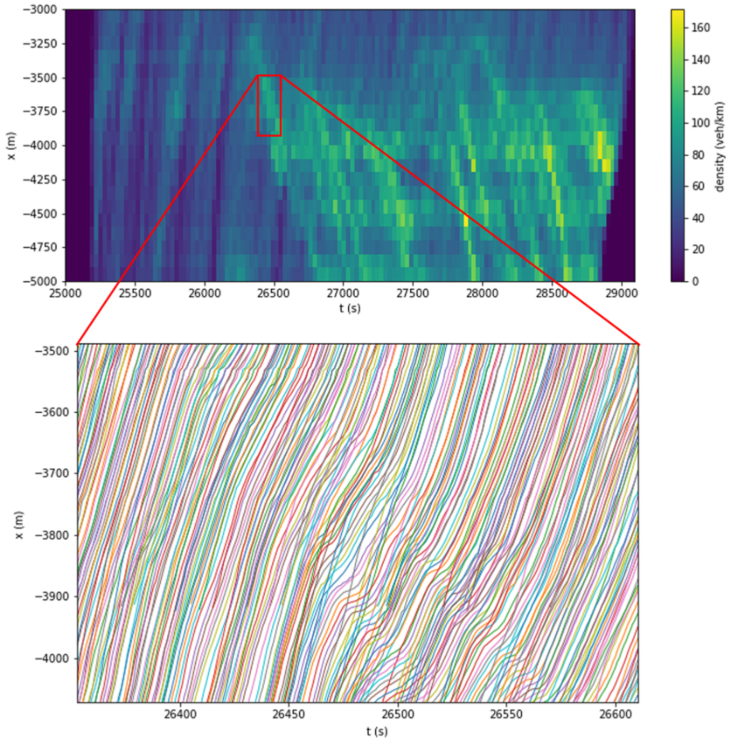


Fig. 12 Complete trajectory dataset obtained by series of video cameras on Hanshin Expressway, Japan (Data from Hanshin Exp. Co. Ltd., 2018). Top: aggregated traffic state. Bottom: trajectories near breakdown.

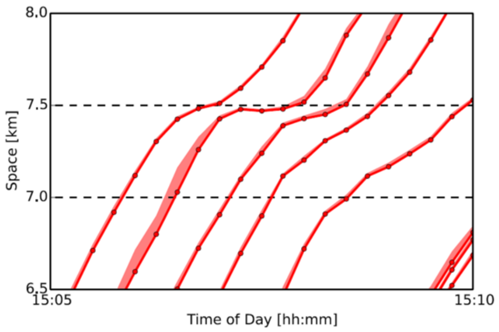


Fig. 13 Spacing data collected by probe vehicles near breakdown on Tokyo Metropolitan Expressway, Japan (Data from Seo et al., 2015). Red curves indicate trajectories of probe vehicles, and red areas indicate spacing between each probe vehicle and its leading vehicle.

Recent advancements in information and communication technology enable us to collect new data that capture continuous spatial-temporal traffic dynamics. Proper use of these data will improve the scientific understanding of breakdown. Notable examples are as follows. First, complete vehicle trajectory data, obtained by fixed cameras and image recognition technology, are useful experimental data. The data captured the complete, continuous spatial-temporal traffic dynamics of specific road sections where the cameras were installed. For example, NGSIM data covers an approximately 600 m highway section for 15 min, and Zen Traffic Data (Hanshin Exp. Co. Ltd., 2018) covers a 2 km highway section for 1 h. An example from Zen Traffic Data is shown in Fig. 12. Second, sampled vehicle trajectory data, obtained by probes or connected vehicles equipped with positioning devices, are becoming common owing to the widespread use of global navigation satellite systems and smartphones (Herrera et al., 2010). Although they do not include complete information on the traffic (i.e., only sampled trajectories and speed), they can cover wide-ranging areas. And third, sampled vehicle trajectory and spacing data, obtained by probe or connected vehicles equipped with ranging devices, will be useful (Seo et al., 2015). Those data capture local density and speed dynamics for wide-ranging areas, from which the complete, continuous traffic state can be estimated. An example of spacing data is shown in Fig. 13.

CAVs will enable breakdown to be controlled. As reviewed in the previous sections, breakdown is likely to be emerging from certain microscopic vehicle behaviors. Thus, by properly designing the driving behavior of CAVs, it may be possible to prevent or alleviate breakdowns. Especially, cooperative driving among CAVs may be useful, because it may enable efficient longitudinal and lateral movements which are impossible for ordinary human-driven vehicles. In-depth understanding of breakdown phenomena will be essential for breakdown prevention by CAVs.

Acknowledgement

This research is partially supported by JSPS KAKENHI Grants (16K18164, 19K04637, 19H02268).

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