The evolution of dynamic ridesharing system based on rational behaviour of users

Dynamic ridesharing system (DRS) is a system where users can find ridesharing partner(s) at any time even shortly before making a trip. The DRS without considering individual preference may cause dissatisfied matchings of users in a shared vehicle and lead to the abandonment of DRS in a long term. To investigate the evolution of DRS, such as long-term adoption, this study develops a model of DRS considering the rational behaviour and learning process of the users. The users' behaviour is considered as travel mode choice and ridesharing partner choice decisions under the expected utility maximisation manner. A day-to-day evolution of a DRS is simulated based on the proposed model, and the effects of users' learning behaviour and some social factors to the long-term DRS adoption are investigated

Keywords: dynamic ridesharing system; behaviour-based model; day-to-day dynamics; passenger matching problem

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1. Introduction

Ridesharing transport is defined as a sharing of car journeys so that more than one person can travel in a single car. It has become attractive alternative as it compromises the advantages between public and private transports. It is expected to mitigate the pollution caused by traffic congestions and reduce resource consumption at the acceptable degradation of travellers' comfort and convenience compared with conventional public transport (Erdoğan, Cirillo, & Tremblay, 2015). Especially, *dynamic ridesharing system (DRS)*, which is a real-time and on-demand ridesharing that can continuously process users appearing in a sequence of time using the advantages of information and communication technology (Agatz et al., 2010), has been broadly studied. For example, if users are effectively assigned to the efficiently operated vehicles, the total vehicle miles can be minimised. Such assignment is considered as social optimum (SO) type of DRS and has been extensively investigated (Agatz et al., 2011). The vehicles operation has also been studied especially in demand responsive transport studies (Karbassi & Barth, 2003).

Aside from SO-based DRS, the user-driven nature in DRS should not be overlooked. In general, ridesharing trip requires at least two users who *desire* to share a ride together. It means that user's itinerary of ridesharing trip (i.e. travel route, travel time) strongly depends on ridesharing partner's behaviour and itinerary. Moreover, user's behaviour could be adapted over days through their experience on DRS performance. This kind of rational behaviour of users are often overlooked by the SO-based DRS studies. For instance, if users can be matched with a desirable partner within an acceptable period of time, users are then willing to continue using DRS. It implies that sufficiently large number of users can convince more users to rideshare—a positive feedback. On the contrary, if users have too high expectation on DRS, users may fail to be matched with a desirable partner within acceptable period of time, so users may not continue ridesharing—a negative feedback. This mixture of positive and negative feedbacks makes the evolution of the DRS complicated and difficult to be predicted by a naïve model (Rohlfs, 1974). Therefore, the methodology to derive DRS's performance and its characteristics based on user's behaviour is necessary.

One of the methodologies to describe aforementioned system's evolution based on user's rational behaviour is day-to-day dynamics model of the system (Mahmassani, 1990). Specifically, the day-to-day dynamics of DRS is an adjustment (i.e. learning) process of users' travel mode choice between ridesharing and others. It can be described by mutual relationship between users' response (e.g. decision on mode choice) and users' experience, expectation, or perceived information (e.g. fare reduction, travel time increase, waiting time, ridesharing partner). However, the day-to-day dynamics of DRS based on users' rational behaviour have not been well studied.

This study aims to investigate the evolution of DRS and its properties based on rational travel mode and partner choices of users. Specifically, this study formulates a behaviour-based DRS model that explicitly describes the mechanism and consequence of the within-day model to the day-to-day model and vice versa based on users' rational behaviour on travel mode and partner choices. The rational behaviour means that user makes decisions by maximising the expected utility. Then, the evolution of the DRS is simulated based on the proposed model. The effects of learning behaviour of users and social conditions to the performance of DRS are investigated.

The remainder of the paper is organised as follows. In section 2, existing studies on ridesharing and day-to-day dynamics are reviewed. In section 3, the model of behaviour-based DRS is formulated. In section 4, the numerical experiments are conducted to investigate the day-to-day dynamics of the formulated model. This paper is concluded in section 5.

2. Literature review

The existing DRS models are reviewed in section 2.1. The concept of day-to-day dynamics is introduced together with the review of day-to-day dynamics in DRS studies in section 2.2. Finally, we summarise what has not been studied and explain the main originality of our study in section 2.3.

2.1. Models for DRS

In this study, the models of DRS can be categorised into two types: *SO-based model*, and *behaviour-based model*, which are reviewed as follows.

2.1.1. SO-based model

Most of the DRS models have been formulated as *SO-based model* with the objective of optimising social cost such as total travel distance minimisation, total travel time minimisation, and number of matching pairs maximisation (Agatz et al., 2011; Agatz et al., 2012; Di Febbraro, Gattorna, & Sacco, 2013; Furuhata et al., 2013; Hosni, Naoum-Sawaya, & Artail, 2014; Li et al., 2014; Shen, Huang, & Zhao, 2016). The SO solution is basically enforced on users by the service providers. This means that individuals' decision-making behaviour is discarded, their choices (e.g. ridesharing partner, driver/rider role) are fully controlled by the DRS service provider, and each user's individual utility is not necessarily improved by using DRS service.

2.1.2. Behaviour-based model

Users' decision-making in DRS (e.g. mode choice, ridesharing partner choice) has been modelled with behavioural assumptions to study the effect of user-driven nature in DRS.

Some studies developed rule-based behaviour model in which users' decision will be made if the specific conditions are satisfied. Fagnant & Kockelman (2015) and Levin et al. (2016) have developed DRS models by assuming that individual trip will be shared if the trip duration does not exceed the specified acceptable threshold, in order to analyse their route assignment problem. In these studies, user's utility is not necessarily maximised, meaning that users are not very rational. Kleiner et al. (2011) have proposed the parallel auction-based DRS that modelled a more rational users' behaviour on ridesharing partner decision where individual makes decision by maximising her/his utility under the restriction of auction procedure. The auction-based model was further employed by Nourinejad & Roorda (2016) to formulate a decentralised behaviour-based DRS.

Other studies have employed the matching theory (Gale & Shapley, 1962) to describe users' rational behaviour when all users try to maximise their utility. In the matching theory, *stable matching solution* is a matching result where no participant has incentive to change her/his matched partner, meaning that it shall be satisfied by completely rational users. Wang (2013) and Yotsutsuji, Sasaki, & Maruyama (2016) adopted a stable marriage problem (Gale & Shapley, 1962) to find a stable matching solution between single driver and single rider in a within-day context of DRS. Thaithatkul et al. (2015a, 2015b, 2016) used modified versions of the stable roommates problem (Knuth, 1997) to represent the static and dynamic matching problems between passengers by considering user preference.

2.2. Day-to-day dynamics

In transport studies, the day-to-day dynamics has been studied to describe the evolution of system (e.g. adjustment of mode choice, route choice) over days (Smith, 1984; Watling, 2003). The evolution of the system is a consequence of users' behaviour and vice versa. To be specific, users adapt their behaviour over days according to their experience on the system's outcome, while the outcome of the system is the result of users' behaviour. Such mechanism can lead the system to evolve to the different state depending on various conditions (Smith et al., 2013).

In ridesharing-related studies, Djavadian & Chow (2016) recently proposed an agent-based day-to-day dynamics of flexible transport service involving ridesharing for evaluating their operating policies. The individual decisions consist of choosing a mode which maximises utility and choosing departure time which minimises schedule delay. As assigning partner to users by dynamic vehicle routing problem is one of their operating policies, the user's behaviour on choosing partner is not considered.

2.3 Discussion

Because of a user-driven nature of DRS, modelling users' behaviour is considerable to describe the DRS performance and its evolution which is a consequence of users' behaviour. Although some studies have modelled DRS considering users' behaviour, their models mainly consider behaviour on ridesharing partner choice in within-day context without explicitly considering the behaviour on mode choice decision. In practice, user's decision in DRS includes both travel mode and partner choices which are strongly dependent upon the user's learning process in day-to-day context.

Moreover, the day-to-day dynamics of DRS is important to be investigated as it can describe the socially important aspects of the system such as the feasibility and sustainability of the DRS. For example, one of the reasons of the decreasing number of carpooling users (American Association of State Highway and Transportation Officials, 2014) can be considered as a day-to-day process where users decide not to use carpooling as their previous carpooling trips did not satisfy their preference. The similar mechanism may exist in DRS. Therefore, to predict the consequence of such mechanism of DRS, day-to-day process of DRS should be modelled considering user's behaviour comprehensively. Moreover, the appropriate control strategies[1] to optimise DRS can be developed based on a day-to-day model.

To fill the research gaps, in this study, user's behaviour in DRS is modelled as an integrated model of within-day and day-to-day models. The user's rational behaviour on travel mode and partner choices in the within-day model is represented by the expected utility maximisation concept because of its high rationality comparing with rule- and auction-based concepts which existing studies employed.

3. Behaviour-based DRS model development

The framework of considering behaviour-based DRS is firstly explained in section 3.1. Then, the behaviour-based DRS is formulated in sections 3.3 – 3.5 under the assumptions explained in section 3.2. The formulation is basically divided into three parts: user's utility function (section 3.3), within-day model (section 3.4), and day-to-day model (section 3.5). The parameters and variables used in the model formulation are listed in Table 1.

3.1. Framework of behaviour-based dynamic ridesharing system

The main components of behaviour-based DRS are users and ridesharing service provider.

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[1] The control strategy is, for example, giving incentive (positive or negative) to users based on difference between solution at stationary state of DRS and SO solution.

Users are travellers who may have willingness to search for partner(s) to share their upcoming trip. Ridesharing service provider offers a service of DRS where users can find partner(s) shortly before making a trip, and operates the system and vehicles.

Users in the DRS make all ridesharing-related decisions by themselves. Once users appear in transport system to make a trip, they firstly make a decision whether to use the DRS—this is a travel mode choice. If users choose the DRS, they can find the *desirable* partner(s) based on their preference—this is a partner choice. The matching result is a spontaneous consequence of every user's utility maximisation. If users are matched with the desirable partner(s), they then make a ridesharing trip. Otherwise, users may stop searching for the desirable partner(s) at any time and choose other travel mode (i.e. waiting choice). These decisions can be conceptually shown in Figure 1, where waiting choice is represented by travel mode choice as it is the decision between ridesharing (i.e. continue using the DRS) and other travel mode (i.e. stop using the DRS).

Ridesharing service provider's roles are: to offer the DRS that facilitates matching process, to operate the vehicles, and to decide fare system. The vehicle operation and fare system determine the cost and benefit of using the DRS from the users' point of view. As the first step of this behaviour-based DRS modelling, the vehicle operation and fare system are not explicitly modelled. The travel time (which is a result of vehicle operation) and fare functions are assumed given.

To model the aforementioned user's decision making process, one has to determine utility of each choice. The utility of ridesharing with *given* partner or other conventional travel modes can be easily determined. Contrarily, the (expected) utility of ridesharing with *unknown* partner, which represents utility of "Rideshare" option in "Mode choice decision", is not obvious because it involves other users' behaviour. The proposed model tackles this problem by introducing the concept of day-to-day process.

3.2. Key assumption

The behaviour-based DRS is formulated under following key assumptions

(a) A mean of transport is a for-hire vehicle with two available passenger seats.
(b) Travel demand is assumed homogenous during a certain time period. Users recurrently travel over days. In each day, users intermittently and randomly appear in the transport system in a sequence.
(c) Expected utility maximisation concept is employed for individual decision-making strategy.

(d) Utility is evaluated by monetary-based factors consisting of cost of in-vehicle travel time, travel fare, and penalty of excessive travel time.

(e) Penalty of excessive travel time is a cost that occurs if user's travel time is longer than her/his acceptable travel time which is a monotonically increasing function of excessive travel time.

(f) Individual expectation-of-utility is determined through day-to-day learning process, which is the weighted sum of private information based on personal memory and the collective information from others.

The process in the model is illustrated in Figure 2. Regarding the assumption (a), travel mode choice is limited to ridesharing and riding alone (i.e. "Yes" and "No" in <Use DRS?> decision node in Figure 2, respectively). A users' partner choice is a one-to-one matching problem for simplicity. The mentioned for-hire vehicle can be, for example, a conventional taxi or an autonomous taxi (Fagnant & Kockelman, 2015).

Regarding the assumption (b), considering the large population, the arrival of users is assumed completely random (Poisson arrival) with the constant arrival rate during a certain time period (e.g. rush hours)[2]. Due to the randomness of users' arrival, users do not know other users' arrival time and travel itinerary in advance. Therefore, a user with rational behaviour makes the decisions by maximising the *expectation*-of-utility under incomplete information in assumption (c).

Regarding assumption (d), in-vehicle travel time of ridesharing is larger than or equal to that of travelling alone, because of the detour. Travel fare is basically deducted by ridesharing. The exact values of the travel time and fare depend on itineraries of the user and partner (Nielsen et al., 2015). Penalty of excessive travel time stated in assumption (e) can be caused by the waiting time for ridesharing partner and/or detour.

User's expectation-of-utility can be updated by learning the DRS over days. Regarding the assumption (f), users are assumed to learn the DRS from two sources: private information and information collected from others, similar to some existing researches on day-to-day adjustment process (Djavadian & Chow, 2016; Iryo, 2016). The private information is what user has privately learned about DRS so far. Specifically, it is the information of her/his latest ridesharing experience blended with her/his memory.

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[2] This means that the model is applied to the analysis of DRS usage for a certain time interval with homogeneous travel demand (e.g. rush hours) rather than the analysis of entire day.

The collective information is supplemental information provided by external sources via any communication channels, such as private communication with others and advertising. The collective information is assumed to be the average performance that users who rideshared in previous day experienced.

3.3. Utility

The utility function $v_{i,k}(j,t)$ of user i ridesharing with user j at time t in day k, where i and j are members in $\boldsymbol{S}_k$, is defined as

$$v_{i,k}(j,t) = -g_i(j) - f_i(j) - d_i(TT_{i,k}(j,t)) \text{ for } \forall i,j \in \boldsymbol{S}_k, \qquad (1)$$

with the travel time

$$TT_{i,k}(j,t) = \tau_{i,k}(t) + x_i(j) \text{ for } \forall i,j \in \boldsymbol{S}_k. \qquad (2)$$

Notice that utility of travelling alone for user i is expressed as $v_{i,k}(i,t)$.

Cost of in-vehicle travel time $g_i(j)$ and travel fare $f_i(j)$ of user i are changed according to in-vehicle travel time involving the necessary detour for ridesharing which directly depends on user j's itinerary. The penalty of excessive travel time $d_i(TT_{i,k}(j,t))$ represents the cost when a user arrives at her/his destination later than desire. The travel time $TT_{i,k}(j,t)$ is period of time from her/his appearance in the transport system to the arrival time to the destination as conceptually shown in Figure 3. The functions $g_i(j)$, $f_i(j)$, $d_i(TT_{i,k}(j,t))$, and $x_i(j)$ are required to be specified by other models explained in the later sections.

3.4. Within-day model

3.4.1. Expectation-of-utility

If users do not know who is the partner (due to the randomness of users' arrival sequence $\boldsymbol{S}_k$), then user's decision will be made by maximising individual *expectation*-of-utility. Following the Equation (1), the expectation-of-utility for ridesharing[3] $EV_{i,k}(t)$ of user i

[3] Expectation-of-utility is not equivalent to the expected utility as travel demand is assumed homogeneous so that the probabilistic aspect of stochastic process is not considered.

at time t in day k is defined as:

$$EV_{i,k}(t) = -EG_{i,k} - EF_{i,k} - d_i(\tau_{i,k}(t) + ETT_{i,k}) \text{ for } \forall i \in \boldsymbol{S}_k. \tag{3}$$

The expectations for ridesharing on cost of in-vehicle travel time $EG_{i,k}$, travel fare $EF_{i,k}$, and travel time $ETT_{i,k}$ are constant in each day k as travel demand is assumed homogeneous, which are realised through a day-to-day learning process explained in section 3.5. The individual expectation-of-utility for riding alone $EV_{i,k}^A(t)$ is assumed to be known equal to $v_{i,k}(i,t)$ as a conventional mode.

3.4.2. Mode choice

The travel mode choice decision of user i at time t is defined as a deterministic function which can be expressed as

$$\phi_{i,k}(t) = \begin{cases} 1 & \text{if } EV_{i,k}(t+\Delta t) > EV_{i,k}^A(t) \\ 0 & \text{if } EV_{i,k}(t+\Delta t) \leq EV_{i,k}^A(t) \end{cases} \text{ for } \forall i \in \boldsymbol{S}_k \tag{4}$$

following the expected utility maximisation concept. Assuming that the matching process is done at every specific time interval Δt to involve newly arrived users, user i uses the DRS (i.e. $\phi_{i,k}(t) = 1$, “Yes” at <Use DRS?> decision node in Figure 2) only if finding a partner in the incoming matching at time $t + \Delta t$ is expected to be better than travelling alone at time t. If so, s/he will then make a partner choice decision at $t + \Delta t$ explained in the following section. Otherwise, user i will travel alone at time t denoted as his exiting time $t_{i,k}^e$. Her/his actual travel mode is denoted as $\phi_{i,k}^e$ which is $\phi_{i,k}(t_{i,k}^e) = 0$ (i.e. “No” at <Use DRS?> decision node in Figure 2). S/he then receives utility at $v_{i,k}(i, t_{i,k}^e)$ where her/his time spent in DRS $\tau_{i,k}(t_{i,k}^e)$ is denoted as $\tau_{i,k}^e$.

3.4.3. Partner choice

A partner choice can be modelled by using any existing behaviour-based model; however, in this study, the static one-to-one passenger matching problem modified from stable roommate problem (Knuth, 1997; Thaithatkul et al., 2015a) is employed to represent a

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However, expectation-of-utility becomes equivalent to the expected utility at the stationary state of the system where users do not change their behaviour over days.

matching process because of its high rationality explained earlier. The employed model can result the stable matching among current users if exists. The stable matching is a set of matching pairs between two users where there are no two users, who are not paired in the stable matching solution, prefer each other to their paired partner. If stable matching is not unique (Irving, 1985), the solution will be the one that provides the maximum total ridesharing utility. In case the stable matching does not exist among any users, those users then do not match with any user but themselves.

The input of the employed model is so-called preference list which is, for example, a list that user i sorts all the existing users in DRS at time $t + \Delta t$ (including user i) corresponding to $v_{i,k}(j, t + \Delta t)$. Under the expected utility maximisation concept, at time $t + \Delta t$, user i prefers to rideshare with any user j if and only if $v_{i,k}(j, t + \Delta t)$ is better than travelling alone $EV^A_{i,k}(t + \Delta t)$ as well as waiting for the next matching $EV_{i,k}(t + 2\Delta t)$. With this users' preference list, the matching solution among existing users can be obtained using the algorithm proposed by Irving (1985).

The solution of matching round r (given that each executing matching at every Δt is called matching round r) is a set of matching pairs where user i matches with partner $m^r_{i,k}$, which can result in two types: (i) $m^r_{i,k} = j$ where $i \neq j$, and (ii) $m^r_{i,k} = i$. Type (i) means a stable ridesharing pair where user i and user j prefer to share a trip to each other. Note that if $m^r_{i,k} = j$ where $i \neq j$ holds, then $m^r_{j,k} = i$ as well. This matching pair always satisfies the condition $v_{i,k}\left(m^r_{i,k}, t + \Delta t\right) \geq \max\{EV^A_{i,k}(t + \Delta t), EV_{i,k}(t + 2\Delta t)\}$. Then $m^r_{i,k}$ becomes an actual partner of user i in day k denoted as $m^e_{i,k}$. So that user i exits the DRS at time $t^e_{i,k}$ and makes a ridesharing trip with utility $v_{i,k}\left(m^e_{i,k}, t^e_{i,k}\right)$; her/his actual mode choice is $\phi^e_{i,k} = 1$. These users who eventually rideshare are called *ridesharing users*. Type (ii) means that there is no other user that user i can be stably paired with. In this case, user i makes a waiting decision represented by Equation (4) where $t = t + \Delta t$.

3.5. Day-to-day model

To process the within-day model, one has to determine values of $EG_{i,k}$, $EF_{i,k}$, and $ETT_{i,k}$. In this study, these expectation variables are determined by the day-to-day model in which a user learns DRS performance following the assumption (f) which can be expressed as

$$EG_{i,k} = \gamma_i \left[\beta_i g_i\left(m^e_{i,K_{i,k}}\right) + (1 - \beta_i) EG_{i,K_{i,k}}\right] + (1 - \gamma_i)\bar{\alpha}_{g,k-1} g_i(i) \qquad \text{for } \forall i \in \boldsymbol{S}_k, \text{(5)}$$

$$EF_{i,k} = \gamma_i \left[\beta_i f_i \left(m^e_{i,K_{i,k}}\right) + (1-\beta_i) EF_{i,K_{i,k}}\right] + (1-\gamma_i)\bar{\alpha}_{f,k-1} f_i(i) \qquad \text{for } \forall i \in \boldsymbol{S}_k, \text{(6)}$$

$$ETT_{i,k} = \gamma_i \left[\beta_i \left(\tau^e_{i,K_{i,k}} + x_i\left(m^e_{i,K_{i,k}}\right)\right) + (1-\beta_i) ETT_{i,K_{i,k}}\right] + (1-\gamma_i)\left[\bar{\tau}_{k-1} + \bar{\alpha}_{x,k-1} x_i(i)\right] \qquad \text{for } \forall i \in \boldsymbol{S}_k. \text{(7)}$$

These mean that a user learns DRS as a weighted sum of private information at weight γ_i and the collective information at weight $1-\gamma_i$ where $0 \le \gamma_i \le 1$. The private information is represented by her/his memory blended with her/his latest ridesharing experience at rate β_i where $0 \le \beta_i \le 1$. The $K_{i,k}$ denotes the day of user i's last ridesharing trip as of day k. The collective information is represented by the average DRS performance of previous day $k-1$ defined as

$$\bar{\alpha}_{g,k} = \frac{\sum_{i \in s_k} \phi^e_{i,k}(g_i(m^e_{i,k})/g_i(i))}{\sum_{i \in s_k} \phi^e_{i,k}}, \tag{8}$$

$$\bar{\alpha}_{f,k} = \frac{\sum_{i \in s_k} \phi^e_{i,k}(f_i(m^e_{i,k})/f_i(i))}{\sum_{i \in s_k} \phi^e_{i,k}}, \tag{9}$$

$$\bar{\tau}_k = \frac{\sum_{i \in s_k} \phi^e_{i,k} \tau^e_{i,k}}{\sum_{i \in s_k} \phi^e_{i,k}}, \tag{10}$$

$$\bar{\alpha}_{x,k} = \frac{\sum_{i \in s_k} \phi^e_{i,k}(x_i(m^e_{i,k})/x_i(i))}{\sum_{i \in s_k} \phi^e_{i,k}}. \tag{11}$$

Note that in order to normalise the absolute difference of ridesharing partner's itinerary, the performance indices $\bar{\alpha}_{g,k}$, $\bar{\alpha}_{f,k}$, and $\bar{\alpha}_{x,k}$ in Equations (8), (9), and (11) are defined as relative values comparing ridesharing trip with that of travelling alone. Meanwhile, the average waiting time $\bar{\tau}_k$ in Equation (10) is directly evaluated. In a case that no one rideshares in day k, it means that DRS is abandoned and its performance is not evaluated.

Based on this learning, users may change their decision over days which could affect the overall DRS average performance. If users learn that they cannot increase their utility by changing their mode choice and do not change their mode choice over days, this state is called stationary state of DRS. Note that the initial condition of the day-to-day dynamics or the expectations for the first day of adopting DRS must be given exogenously.

3.6. Discussion

In order to complete the proposed model, $x_i(j)$, $f_i(j)$, $g_i(j)$, and $d_i\big(TT_{i,k}(j,t)\big)$ have to be specified. They can be specified by employing certain types of models: traffic flow and vehicle operation model for $x_i(j)$, fare system for $f_i(j)$, personal perception on sharing a private space during a trip with other for $g_i(j)$, and schedule late cost for $d_i\big(TT_{i,k}(j,t)\big)$. In the numerical example in section 4, we consider a simple specification to demonstrate the model's basic features explained in section 4.2.

The model parameter Δt has to be specified by service provider. If Δt is specified to be small, user's waiting time for ridesharing partner is expected to be shorter than larger Δt as newly arrived users can be quickly involved in the matching process. On the other hand, if Δt is specified to be large, users could be matched with a more desirable partner as there are more choices comparing with small Δt.

4. Numerical experiment

The objectives of numerical experiments are firstly explained in section 4.1. The proposed model is specified for numerical experiments in section 4.2. The experimental settings are described in section 4.3. Then the results are explained in section 4.4 and discussed in section 4.5.

4.1. Objectives of numerical experiment

The numerical experiments are conducted to quantitatively investigate the evolution of DRS under different user's learning behaviour and specified social factors and to obtain meaningful insights of DRS. Even though some of the characteristics can be straightforwardly understood from the qualitative comprehension (e.g. low demand leads to failure of DRS), some aspects can hardly be determined without numerical experiments due to the general complexity of DRS explained earlier (e.g. what is the minimum demand (critical mass) to sustainably operate DRS?).

4.2. Model specification for numerical experiments

The components of utility in Equation (1) (i.e. $x_i(j)$, $f_i(j)$, $g_i(j)$, and $d_i\big(TT_{i,k}(j,t)\big)$) are specified as follows. A ridesharing trip between two travellers can consist of three durations (Figure 4): detouring for picking up partner $a_{1,i}(j)$, riding with partner $b_i(j)$, heading to destination after dropping off partner $a_{2,i}(j)$. The travel time of these three durations are given proportional to the Euclidian distance which are dependent on itineraries of user and partner. Assuming that vehicles are effectively operated, vehicles' dispatching time is neglected and traffic is free-flowing. The $x_i(j)$ can then be expressed as

$$x_i(j) = a_{1,i}(j) + b_i(j) + a_{2,i}(j). \tag{12}$$

The fare is considered to be equally shared with partner for the duration of ridesharing. Given that α is a fare rate for one unit of travel time, a fare for the entire ridesharing trip is obtained as

$$f_i(j) = \alpha\big(a_{1,i}(j) + a_{2,i}(j)\big) + (\alpha/2)b_i(j). \tag{13}$$

Given that the costs of in-vehicle travel time during riding alone and ridesharing for one unit of time are different and denoted as μ_1 and μ_2, respectively, the cost of in-vehicle travel time for the entire trip $g_i(j)$ is expressed as

$$g_i(j) = \mu_1\big(a_{1,i}(j) + a_{2,i}(j)\big) + \mu_2 b_i(j). \tag{14}$$

The μ_2 can be larger than μ_1 due to, for example, the discomfort of sharing private space when ridesharing.

The penalty of excessive travel time is specified similar to Arnott, De Palma, & Lindsey (1999) as

$$d_i\big(TT_{i,k}(j,t)\big) = \mu_1(TT_{i,k}(j,t) - TT^*_{i,k})^2, \tag{15}$$

where $TT^*_{i,k}$ denotes the acceptable travel time.

The parameters are specified as follows. The model parameter Δt is given at one unit of time. This value seems fair enough when there is approximately one user appearing in transport system at every one unit of time ($\lambda_S = 1$) as it means that the matching is processed at every time new user is expected to use the DRS. The in-vehicle travel time for ridesharing is considered to cost 50% more than that of riding alone similar to the investigation from Hunt & McMillan (1997). Given that the cost of in-vehicle travel time for riding alone is at 0.1 unit of money per one unit of time ($\mu_1 = 0.1$), ridesharing then costs 0.15 unit of money ($\mu_2 = 0.15$). Note that the larger μ_2, the lower the DRS adoption level is. Such effect becomes less significant when average travel distance of all users becomes shorter (see the result of sensitivity analysis in Appendix). The fare for travelling one unit of time is given to be 10 times larger than the cost of in-vehicle travel time for riding alone ($\alpha = 1$) similar to taxi fare in Tokyo, Japan comparing with its average cost of in-vehicle travel time (Kato, Tanishita, & Matsuzaki, 2010). Lastly, the acceptable travel time $TT^*_{i,k}$ is given to be 10% larger than travel time without using DRS.

4.3 Numerical experiment settings

4.3.1. Numerical experiment design

Users' OD pattern, arrival rate, and users' learning behaviour are considered as input parameters. Each input parameters setting is called scenario. For each scenario, the experiment is conducted for five replications to obtain the average results. Specifically, the experiment on each scenario is conducted for five times on different sets of users generated from random seeds, where one set contains 500 users travelling from many origins to many destinations in two-dimensional city. For each replication, these 500 users continue visiting the transport system with the same OD pattern but random arrival over one thousand days. This design is conceptually visualised in Figure 5 where each grey shaded row represents one replication.

The results are evaluated at the *near*-stationary state instead of the stationary state as the stationary state is difficult to be reached in the numerical experiment as well as in practice. The near-stationary state is defined using the limit of sequence concept similar to some existing day-to-day dynamics studies (Bie & Lo, 2010)—namely, a state of DRS at a certain day is near-stationary if the number of ridesharing users converges to any point within the specified measure of closeness for recent one hundred consecutive days. The measure of closeness is given at $\pm 5\%$ of total users. The number of ridesharing users (i.e. $\sum_{i \in \boldsymbol{S}_k} \phi_{i,k}^{e}$) and long-term DRS performance (i.e. $\bar{\alpha}_{g,k}$, $\bar{\alpha}_{f,k}$, $\bar{\tau}_k$, $\bar{\alpha}_{x,k}$, and total vehicle miles travelled (VMT)) are evaluated at the near-stationary state if exists (red shaded solid boxes in Figure 5). Besides that, the DRS evolution is obtained by averaging the number of ridesharing users for each day k of all replications (blue shaded dashed boxes in Figure 5). The DRS performance of each day k is only evaluated among users whose arrival time $t_{i,k}^{a}$ is within $[0.1T_k, 0.9T_k]$, where T_k is the total time steps that all 500 users appear in transport system. This is to avoid the effects of insufficient users and time caused by beginning and ending period of a finite length day.

4.3.2. Input scenarios

The scenarios on social factors are given as follows. Five scenarios of OD patterns are considered to represent several land use patterns (Figure 6). The origins and destinations are independently and randomly sampled by the multivariate uniform distributions with the same radius of 100 units of distance. In other words, a user's trip is generated by connecting origin and destination randomly. The OD pattern 1 represents the commuting travel pattern where the origins and destinations are distributed over the same centre. The rest OD patterns 2 – 5 are where the centres of destination's distribution are 50, 100, 150, and 200 units of distance away from the centre of origin's distribution, respectively. The averages distance between OD pairs for OD patterns 1 – 5 are 89.44, 99.98, 128.20, 167.50, and 212.60, respectively. Users' arrival rate is presented by average travel demand within one unit of time λ_S as it is assumed to be completely random (Poisson arrival). Six values of λ_S are considered as follows: 0.5, 1, 2, 3, 4, and 5 to represent the increasing travel demand, which also imply the increasing number of partner choices in each matching round.

Regarding users' learning behaviour, all users are assumed to learn the DRS from their private experience at the same learning rate and update rate of memory, so that $\gamma_i = \gamma$ for $\forall i \in \boldsymbol{S}_k$ and $\beta_i = \beta$ for $\forall i \in \boldsymbol{S}_k$, respectively.

The initial conditions of DRS performance for all scenarios are given at the best conditions where users can immediately share their entire trip with ridesharing partner as follows: $\bar{\alpha}_{g,0} = 1.5$, $\bar{\alpha}_{f,0} = 0.5$, $\bar{\tau}_0 = 0$, and $\bar{\alpha}_{x,0} = 1$. It is expected that such initial state is not stable; the number of ridesharing users will change over days and may converge to the near-stationary state if exists. Note that the dependency of DRS's evolution and its near-stationary state on the initial conditions has been tested to be not very strong.

4.4. Results

Firstly, the effects of users' learning behaviour to the DRS's adoption and evolution are presented in section 4.4.1. The DRS evolution for different social factors are presented in section 4.4.2 with the detailed DRS performances at its near-stationary state in section 4.4.3. Lastly, the spatial characteristics of ridesharing users are investigated in section 4.4.4.

4.4.1. Effects of users' learning behaviour

The effects of different users' learning behaviour to the DRS in term of number of ridesharing users at the near-stationary state are shown in Figure 7 for OD patterns 1, 3, and 5 with λ_S at 1 user/unit of time. DRS tends to have larger adoption when users learn more from collective information (lower γ). However, the case when excessive learning from collective information can result in lower adoption level is found in OD pattern 1 (Figure 7 (a)). The results also show the positive effects of individual memory where learning more from their memory results the larger adoption of DRS (lower β). On the other hand, if users learn only from their latest ridesharing experience (without learning from individual memory and collective information; $\beta = 1, \gamma = 1$), DRS turns to be abandoned as no one continues ridesharing.

The evolution when users learn DRS from collective information at different learning rate γ when fixing β at 0.8 is shown in Figure 8 for OD pattern 1 with λ_S at 1 user/unit of time. According to this Figure, $\gamma = 0.8$ has the largest number of users at the near-stationary state. The underlying mechanism could be explained as follows. Without learning from collective information ($\gamma = 1$), users gradually stop ridesharing, and DRS is eventually abandoned. On the other hand, for excessive learning from collective information ($\gamma \leq 0.5$), some of users whose expectations are substantially degraded by the provided collective information do not continue ridesharing.

4.4.2. Effects of OD pattern and demand to overall dynamics

The average numbers of ridesharing users at the near-stationary state for all social factors scenarios are shown in Figure 9. Regarding the learning parameters, for both cases (upper: $\gamma = 0.5$ and $\beta = 1$, lower $\gamma = 0.5$ and $\beta = 0.8$), DRS of the specific scenario of OD pattern and travel demand evolves to the similar near-stationary state.

The DRS adoption level increases from OD patterns 1 to 5. It means that DRS will be frequently utilised if direction of trips is positively correlated, which makes sense. Regarding demand rate λ_s, the DRS adoption level increases as demand increases, which also makes sense. In some scenarios number of users is zero, meaning that the DRS was eventually abandoned. The abandonment of DRS occurs when λ_S is 0.5 users/unit of time for OD patterns 1 and 2. This demand level corresponds to the critical mass in DRS.

4.4.3. Detailed performance of DRS at the near-stationary state

The detailed performance of DRS at the near-stationary state for the case $\beta = 0.8$ is evaluated. Specifically, the average outcomes of all ridesharing users were calculated as a performance indices from view point of users: the relative values of in-vehicle travel time $\bar{\alpha}_{x,k}$ (Figure 10), travel fare $\bar{\alpha}_{f,k}$ (Figure 11), cost of in-vehicle travel time $\bar{\alpha}_{g,k}$ (Figure 12), and the absolute average value of waiting time $\bar{\tau}_k$ (Figure 13). Additionally, the social cost is evaluated by the VMT (Figure 14).

According to the results, ridesharing users have shorter detour (Figure 10) from OD patterns 1 – 5 because of the higher correlation of trips' direction. Figure 11 shows that ridesharing users in high correlated OD pattern can enjoy more travel fare reduction as the trip can be mostly shared with partner (high $b_i(j)$), and only small detour is required (low $a_{1,i}(j)$, $a_{2,i}(j)$). Corresponding to Equation (14), the higher duration of riding with partner $b_i(j)$, the higher cost of in-vehicle travel time is (Figure 12). When ridesharing users perceive that they can reduce big amount of fare by ridesharing with a partner who has very similar itinerary, ridesharing users tend to spend longer time to find that desirable partner (Figure 13). That desirable partner can be found faster when travel demand is increased. In term of social cost, the total VMT are always reduced by DRS (solid lines in Figure 14) comparing to the cases without DRS (dashed lines). The larger the demand rate or the longer the travel distance, the larger the reduction in total VTM is.

4.4.4. Ridesharing user's spatial characteristic at the near-stationary state

Figure 15 shows stacked distribution of users according to their regular travel distance categorised by travel modes: ridesharing (red) and riding alone (blue). The results show that travellers who have long travel distance tend to rideshare. This could be because they can be highly advantaged by travel fare reduction from ridesharing. This conforms the results of larger number of ridesharing users in OD pattern with higher average travel distance and similarity of itinerary in Figure 9.

4.5. Discussion

According to the results, we observed the emergence of important behaviours in DRS because of individual traveller's rational decision making. For example, critical masses of travel demand rate to sustainably operate DRS were found (Figure 9). The size of critical mass strongly depends on spatial distribution of OD demand. Additionally, the users' learning behaviour was observed to have significant effects on the service level of DRS (Figures 7 and 8). It suggests the need in the investigation of optimal collective information provision for each area and the awareness of the collective information propagation as users' learning behaviour can hardly be controlled in practice.

Moreover, we confirmed that the proposed DRS model showed reasonable behaviours that are qualitatively consistent with the common knowledge in the literature. It implies that the proposed model could be useful to quantify these behaviours through simulation-based case studies. For example, DRS tends to be utilized if demand level was high and OD pattern was correlated (Figure 9), reduction in total VMT by DRS increases as demand level increases (Figure 14), a traveller with long travel distance tends to use DRS (Figure 15).

5. Conclusion

This study investigated how dynamic ridesharing system (DRS) evolves in the long term under the rational travel mode and ridesharing partner choices behaviour of travellers. A model of behaviour-based DRS was formulated to present the interrelationship between users' decisions in within-day model and their learning process in day-to-day model. The users' rational behaviours on travel mode and ridesharing partner choices decisions were formulated under the expected utility maximisation concept. The quantitative characteristics of the proposed model were analysed through the numerical experiments to obtain the meaningful insights of DRS.

From the numerical experiments, the effects of learning behaviour and social factors to the DRS's adoption level and its long-term characteristics were investigated. Traveller's learning and information collection behaviours on DRS's performance showed complex effect to DRS's evolution. This highlights the importance of such factors when implementing the DRS. The existence of critical mass in DRS was also confirmed, meaning that there is a minimum required demand to sustainably operate DRS; if this requirement isn't met, DRS will be completely abandoned. The model showed reasonable behaviours that are consistent with qualitative common knowledge. This implies that the model could be useful to quantify such knowledge.

One of the future research directions could be extending our developed behaviour-based DRS by incorporating the system operational models (e.g. vehicle operation, fare system) or other control strategies (e.g. DRS encouragement policy) from DRS service provider side. Investigating the long-term performance of such extended models using the same framework of this study will be considerable as it would provide some useful policy implications. Case studies based on real-world data would be considerable as well. Finally, the numerically investigated long-term characteristics of DRS also need to be analytically proven in future.

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Appendix: Result of sensitivity analysis of ridesharing-related parameter

Figure A1 shows the number of ridesharing users at near-stationary state when cost of in-vehicle travel time for ridesharing for one unit of time μ_2 is given between $[0.1,0.2]$. This shows the cost of in-vehicle travel time for ridesharing varying from the case when it is assumed equal to that of travelling alone to the case when it is doubled.

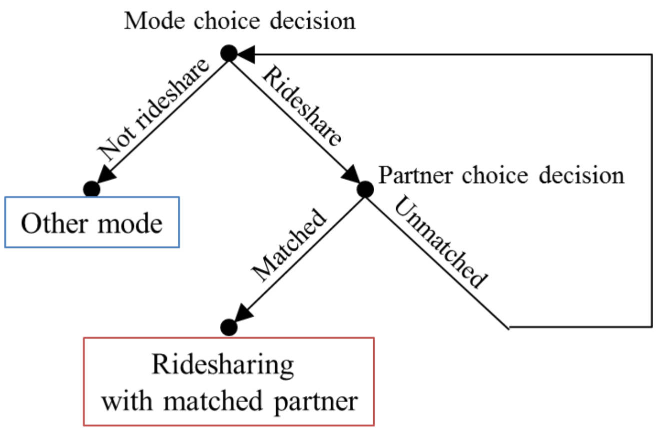


Figure 1. Individual within-day ridesharing-related decision making process.

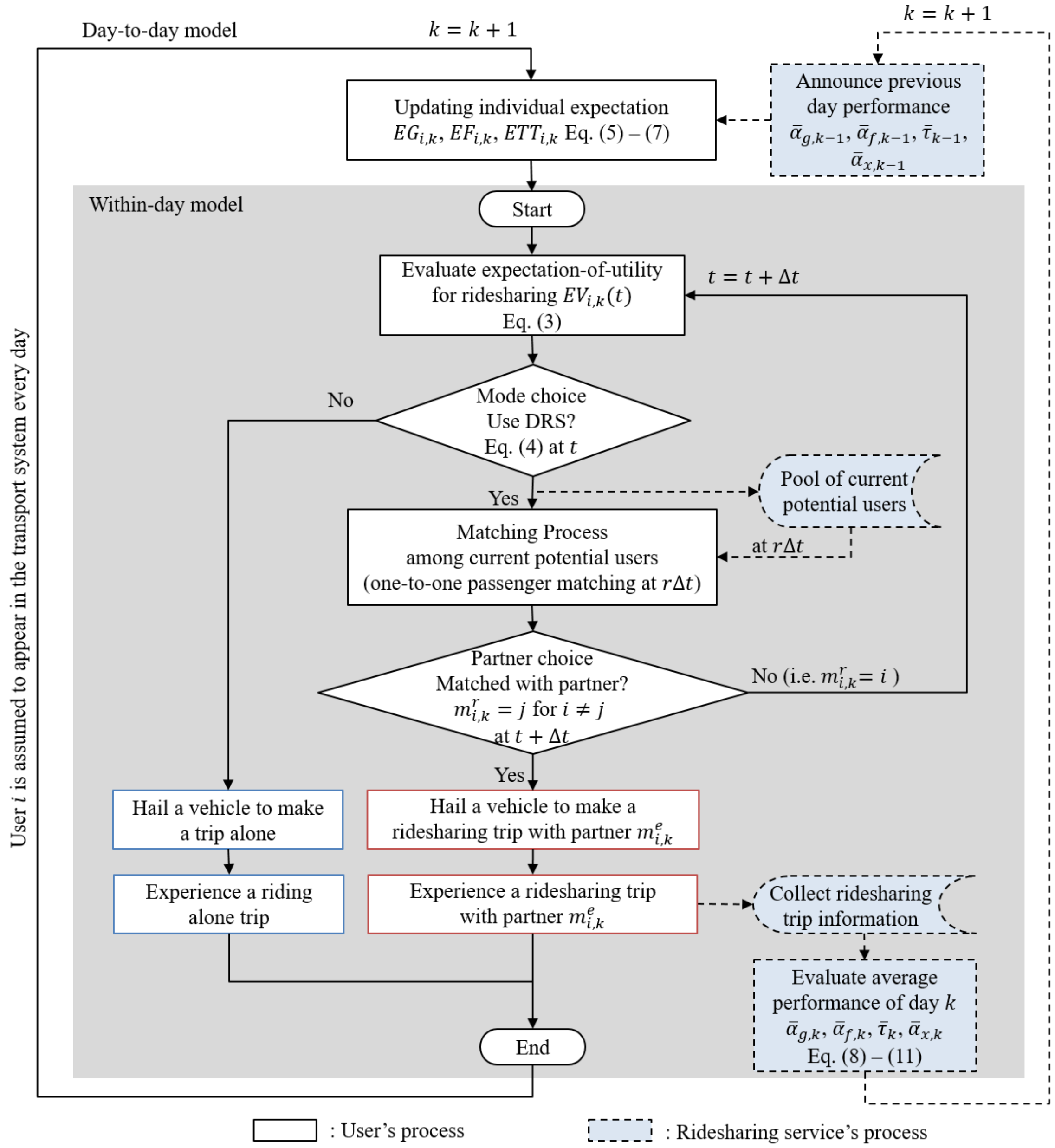


Figure 2. Process in the formulated behaviour-based DRS model.

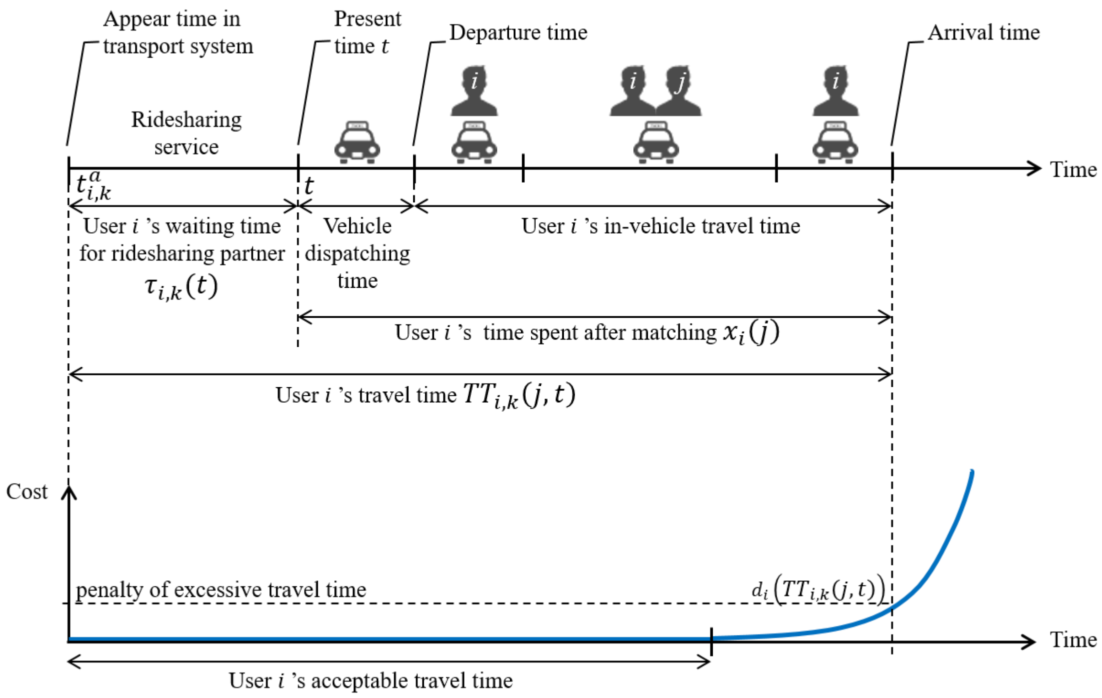


Figure 3. Conceptual illustration of time-related variables and penalty of excessive travel time of user i.

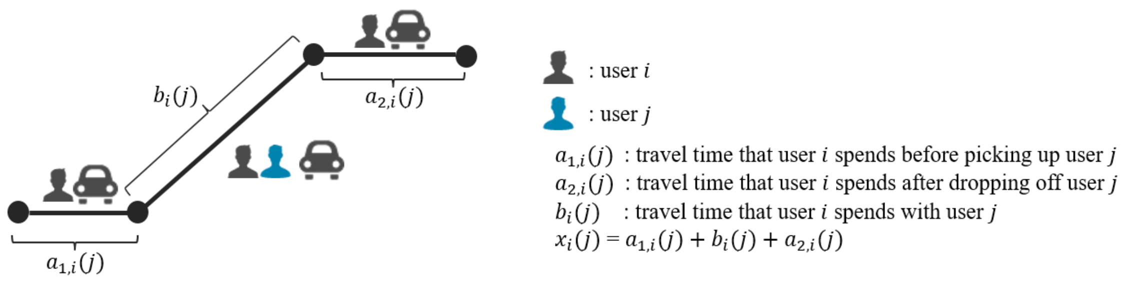


Figure 4. Conceptual illustration of in-vehicle travel time variables used in numerical experiments.

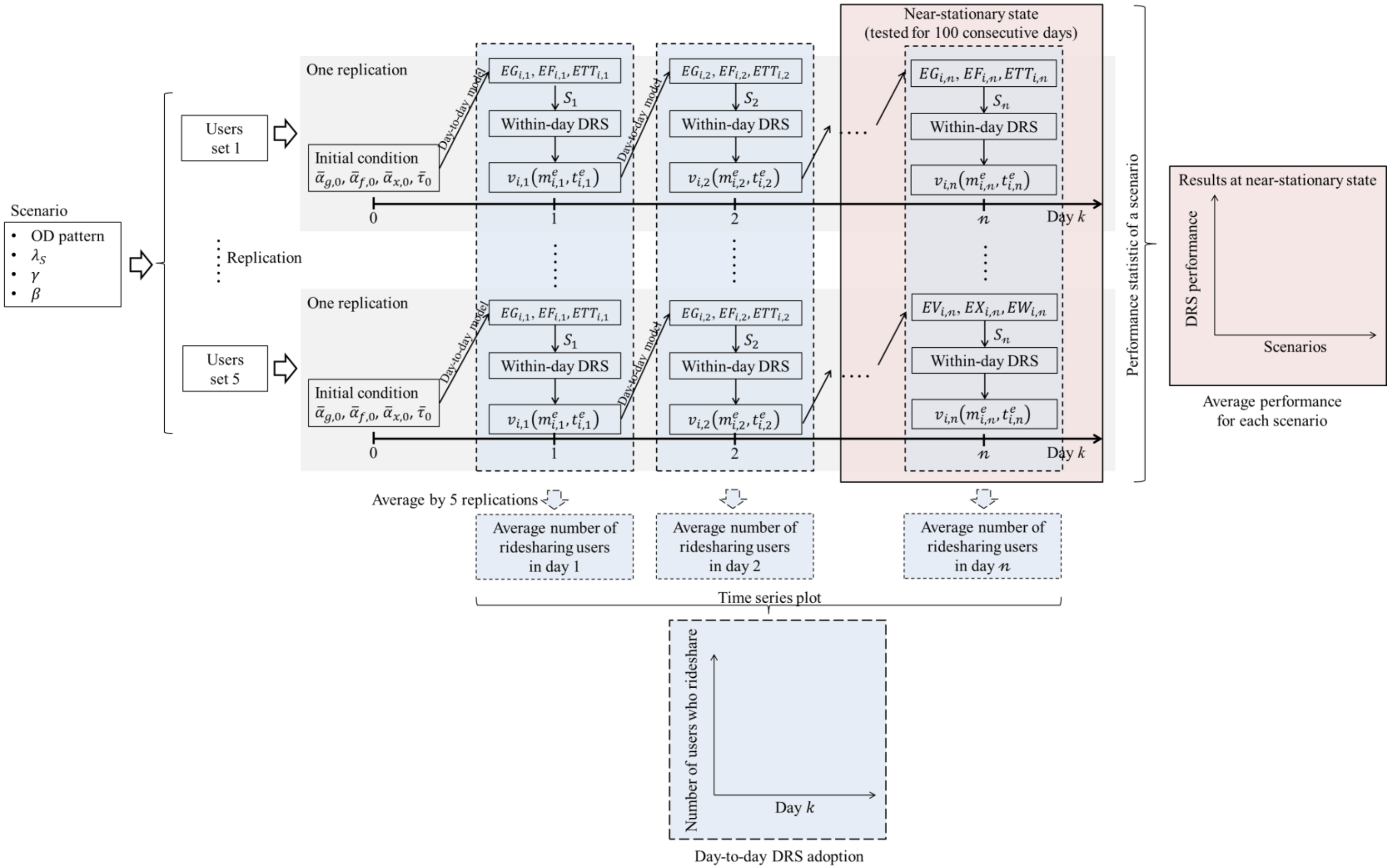


Figure 5. Numerical experiments design.

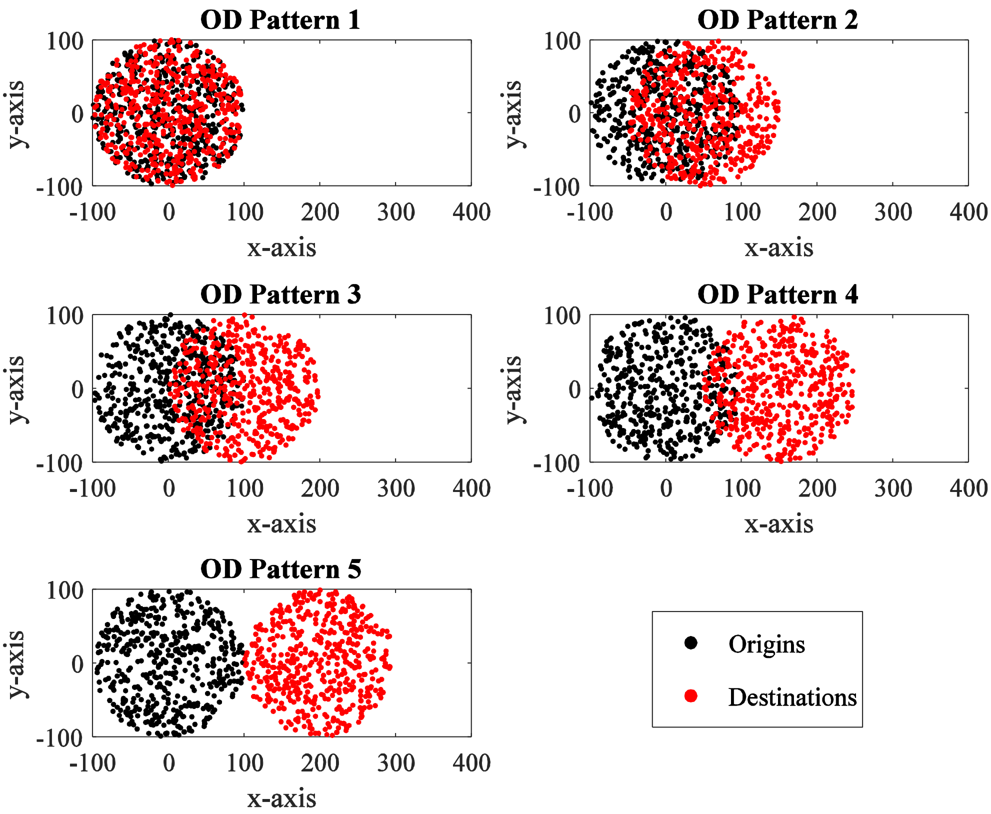
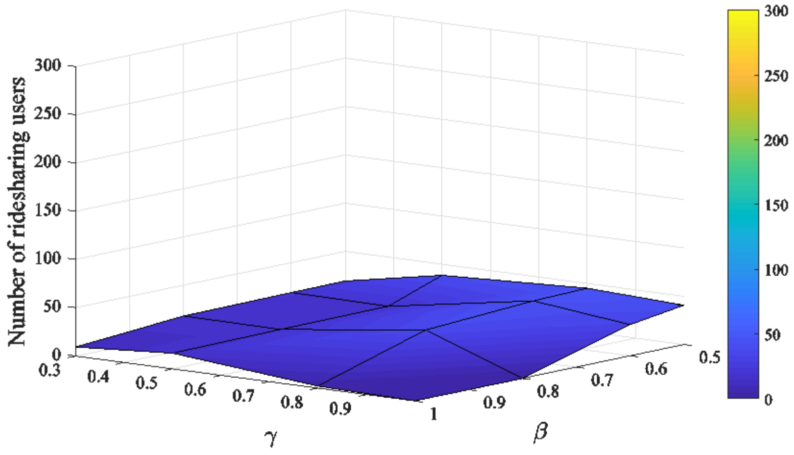
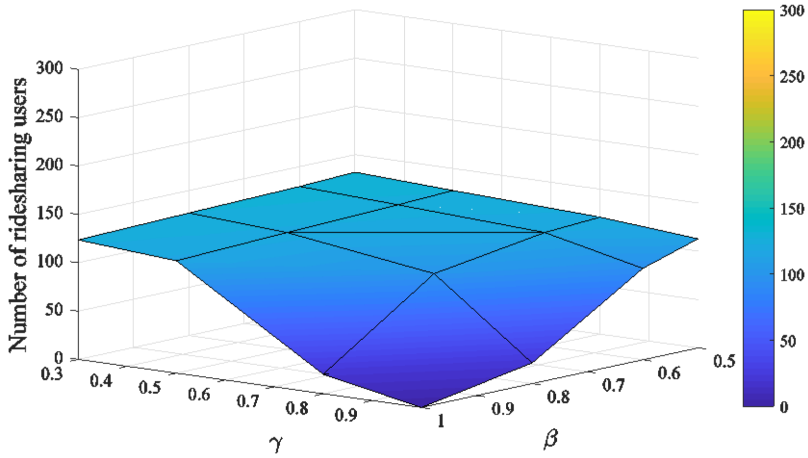


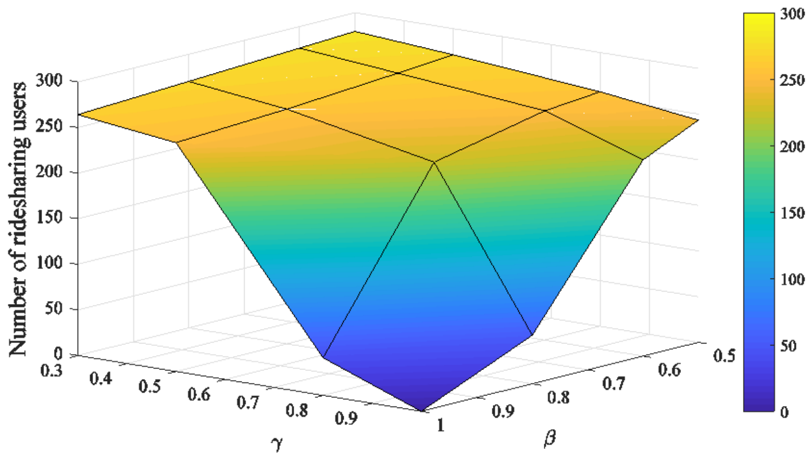
Figure 6. Examples of origins and destinations for five given OD patterns.



(a) OD pattern 1, $\lambda_S = 1$

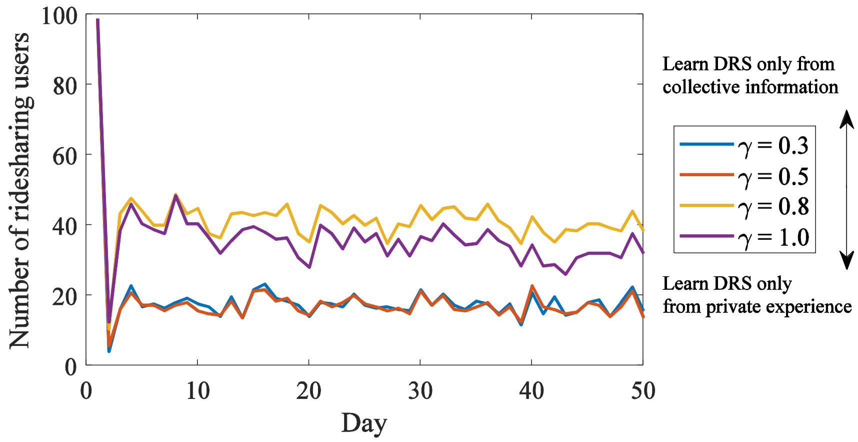


(b) OD pattern 3, $\lambda_S = 1$

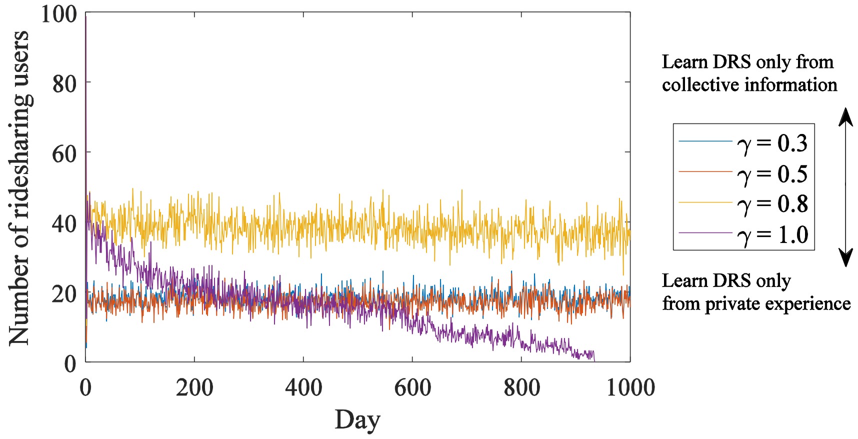


(c) OD pattern 5, $\lambda_S = 1$

Figure 7. Average number of ridesharing users at the near-stationary state for the different users' learning behaviour.

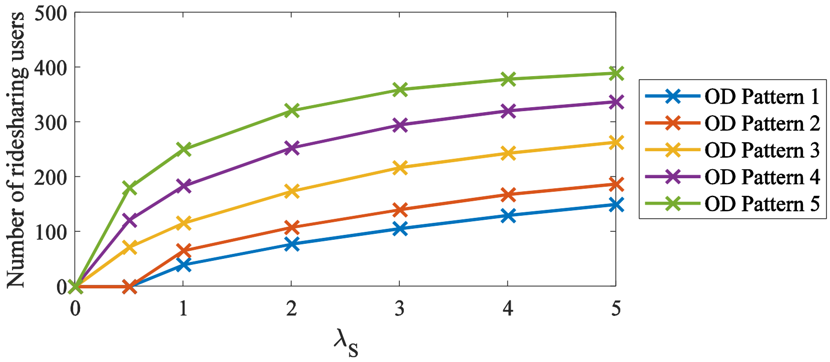


(a) Evolution of first 50 days

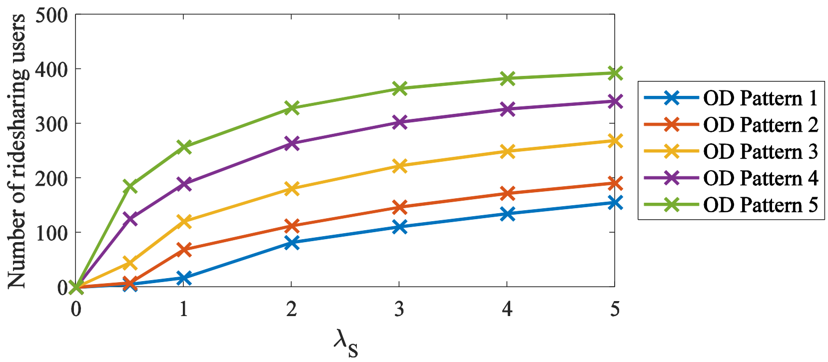


(b) Evolution of 1000 days

Figure 8. Evolution of DRS in term of number of ridesharing users for scenario with OD Pattern 1 and $\lambda_S = 1$.



(a) $\gamma = 0.5$ and $\beta = 1.0$



(b) $\gamma = 0.5$ and $\beta = 0.8$

Figure 9. Average number of ridesharing users at the near-stationary state.

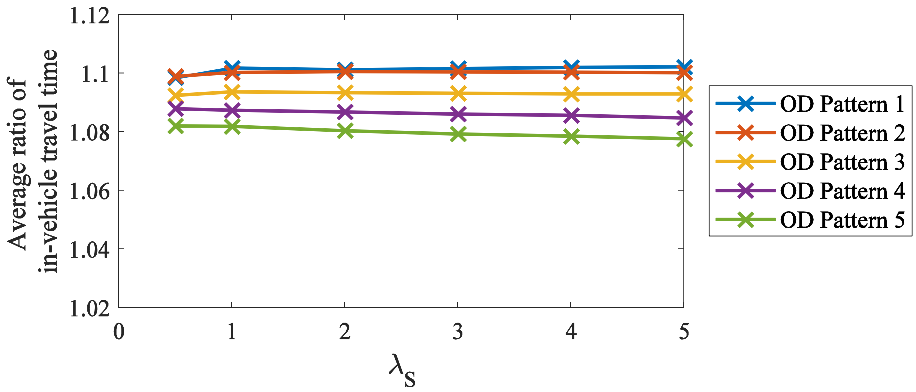


Figure 10. Average ratio of in-vehicle travel time for ridesharing ($\bar{\alpha}_{x,k}$) at the near-stationary state where $\gamma = 0.5$ and $\beta = 0.8$.

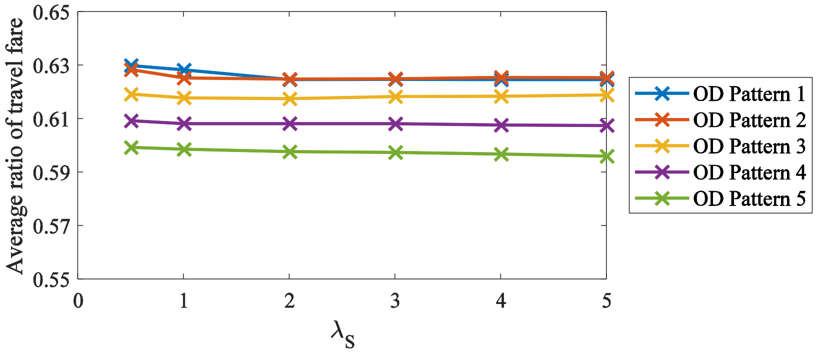


Figure 11. Average ratio of travel fare for ridesharing ($\bar{\alpha}_{f,k}$) at the near-stationary state where $\gamma = 0.5$ and $\beta = 0.8$.

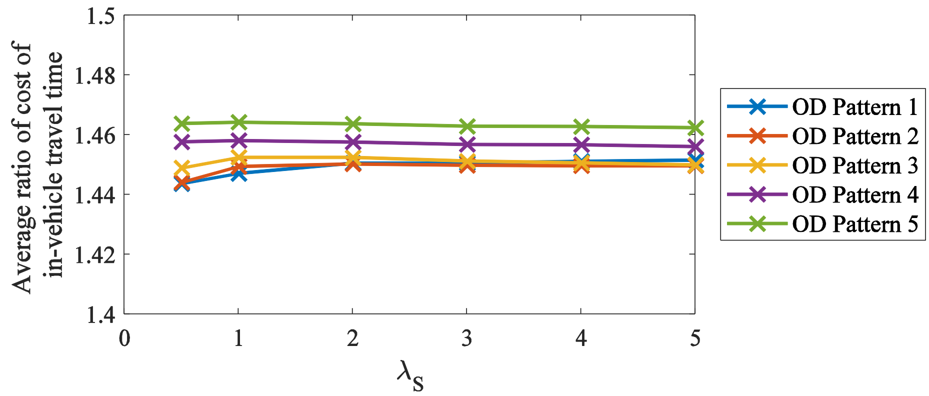


Figure 12. Average ratio of cost of in-vehicle travel time for ridesharing ($\bar{\alpha}_{g,k}$) at the near-stationary state where $\gamma = 0.5$ and $\beta = 0.8$.

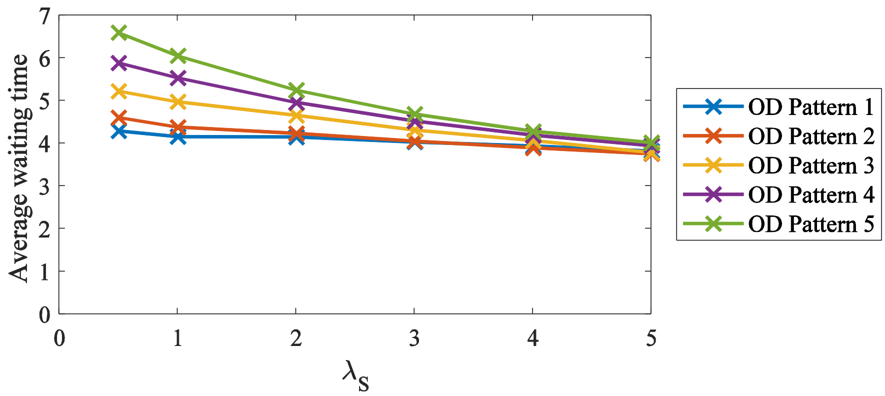


Figure 13. Average waiting time for ridesharing ($\bar{\tau}_k$) at the near-stationary state where $\gamma = 0.5$ and $\beta = 0.8$.

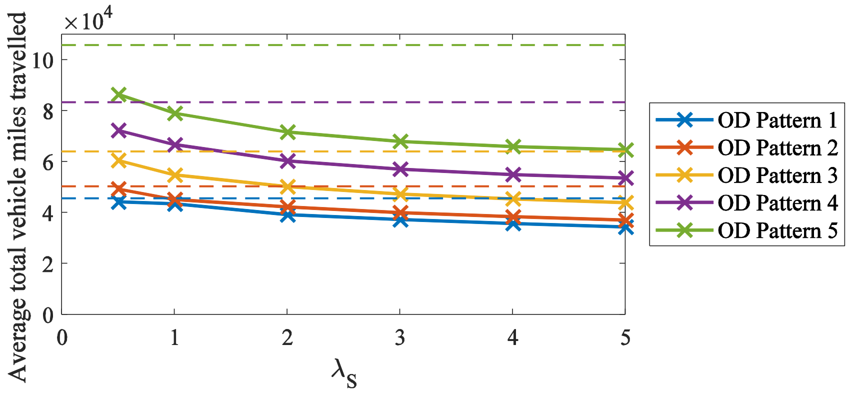


Figure 14. Average total vehicle miles travelled (VMT) at the near-stationary state (solid and dashed lines show the VMT of scenarios with and without DRS, respectively) where $\gamma = 0.5$ and $\beta = 0.8$.

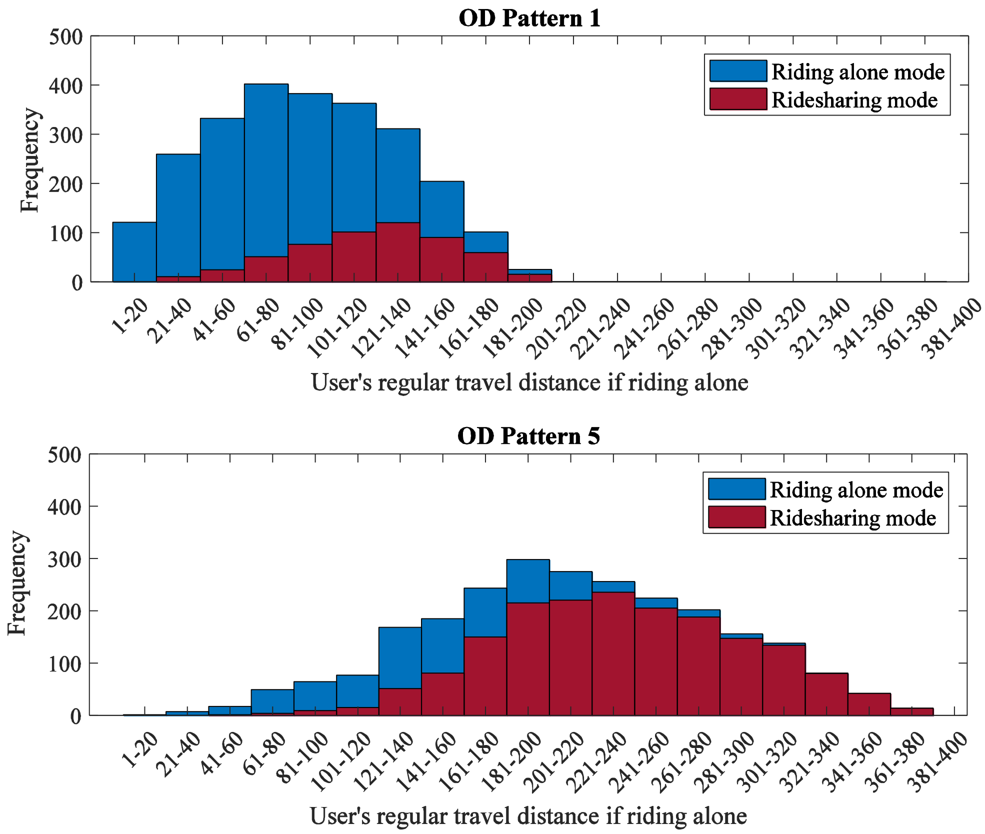


Figure 15. Travel mode share of users grouped by user's regular travel distance at near-stationary state of scenarios where $\lambda_S = 3$, $\gamma = 0.5$ and $\beta = 0.8$.

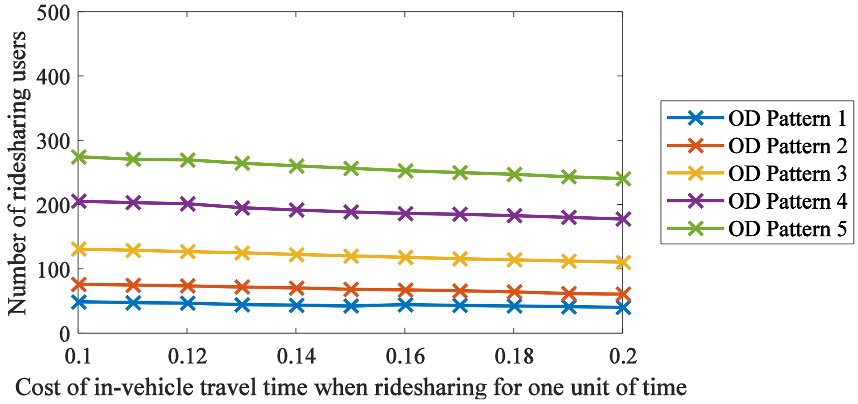


Figure A1. Average number of ridesharing users at the near-stationary state for the different cost of in-vehicle travel time when ridesharing for one unit of time where $\lambda_S = 1$.