Multi-objective linear optimization problem for strategic planning of shared autonomous vehicle operation and infrastructure design

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***Abstract*—This study proposes a unified optimization framework for strategic planning of shared autonomous vehicle (SAV) systems that explicitly and endogenously considers their operational aspects based on macroscopic dynamic traffic assignment. Specifically, the proposed model optimizes fleet size, road network design, and parking space allocation of an SAV system with optimized SAVs' dynamic routing with passenger pickup/delivery and ridesharing. It is formulated as a multi-objective optimization problem that simultaneously minimizes total travel time of travelers, total distance traveled by SAVs, total number of SAVs, and infrastructure construction cost; thus, both the user-side cost and the system-side cost are taken into account, and their trade-off relations can be explicitly investigated. Furthermore, the problem is formulated as a linear programming problem, making it easy to solve. By leveraging the linearity, we mathematically derive a useful property of the problem: introduction of ridesharing can weakly monotonically and simultaneously decrease the user-side cost and system-side cost. The proposed model is evaluated by applying it to actual travel records obtained from New York City taxi data.**

***Index Terms*—dynamic SAV assignment, ridesharing, fleet size optimization, network design, parking space allocation**

I. Introduction

SHARED autonomous vehicle (SAV) systems may be an efficient transportation mode in the future [1], [2]. In an SAV system, automated vehicles shared by a society may transport travelers using optimized routes and/or ridesharing matching. Thus, they may reduce the number of vehicles and infrastructure requirements (e.g., road width, parking space) in a city without sacrificing the travelers' utility.

The design of SAV systems involves solving various types of problems in various levels. In long-term strategic levels, fleet sizing [1], [3]–[5], road network design and autonomous vehicle lane deployment [6], and parking space allocation [7] need to be solved. These problems are important as their solution may have strong impact to the entire society.

In short-term operational levels, the vehicle routing problem with pickup and delivery with time windows (VRPPDTW) [8], [9] and dynamic ridesharing matching [9]–[12] need to be solved. These problems are important as they will allow us to take full advantage of the capabilities of SAVs.

These strategic and operational problems are essentially interrelated, although they have usually been solved separately the literature. It would be favorable if strategic decision making of SAV systems explicitly and endogenously considers operational aspects of SAV systems. For example, road network design for a city with SAV systems will be significantly efficient if it takes SAVs' ridesharing and driver-less parking capability into account, because ridesharing can reduce overall traffic volume, and driver-less parking makes parking slots far from offices/homes more convenient. A unified optimization problem that simultaneously solve these strategic and operational problems would be useful for strategic planning of SAV systems.

The importance of trade-off relations among performance indexes of SAV systems has been noted, especially in strategic levels [13]–[15]. For example, an SAV system could be designed to minimize either user-side cost (e.g., passenger travel time), system-side cost (e.g., operational cost), or social-side cost (e.g., environmental cost); these cases may have completely different system design and cost allocation [13], [16].[1] To determine SAV systems' operator and operational scheme that satisfy a society's political/strategic goals, it would be necessary to explicitly consider these trade-off relations during the strategic planning phase of SAV systems. Such trade-off relations can be explicitly investigated by using the framework of multi-objective optimization problems (MOOP) [17]; however, to the authors' knowledge, application of MOOP to SAV system modeling is very limited.

This study proposes a unified MOOP framework for strategic planning of SAV systems (e.g., fleet sizing, infrastructure design/update) that explicitly and endogenously considers dynamic operational aspects of the systems (e.g., routing and ridesharing). It jointly optimizes aggregated variables on the SAV's dynamic routing with passenger pickup/delivery, dynamic ridesharing, fleet sizing, road network design, and parking space allocation. The objective functions of the MOOP are total travel time of travelers, total distance traveled by

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[1] This issue is related to the type of operators of SAV systems. For example, an SAV system may be operated by a public sector or a private one. In the former case, the objective of the operator may be the maximization of the social welfare (e.g.,the sum of user-side cost, system-side cost, and social-side cost). In the latter case, the objective may be the profit maximization (e.g., passenger fee minus system-side cost).

SAVs, total number of SAVs, and infrastructure construction cost; thus, both of the user-side cost and the system- or social-side cost are taken into account. The traffic and passenger flows in the model is described by a dynamic traffic assignment (DTA) framework; thus, dynamical aspects of SAV operation are considered. The problem is formulated as linear programming, making it easy to solve. Furthermore, by leveraging the linearity of the problem, we mathematically derive a useful property of the problem: introduction of ridesharing weakly monotonically decreases the user-side cost and the system-side cost. These features would be useful for strategic optimization of SAV operation and infrastructure design.

Note that the proposed problem is not intended to optimize actual, detailed SAV operation on a particular day because of its macroscopic nature. Instead, it is intended to serve as a benchmark of operational performances during strategic planning phases.

As mentioned, one of the unique features of the proposed method is that it is computationally efficient to solve. In the existing studies, the optimization problems of an SAV system element were often formulated by using computationally costly frameworks such as mixed integer programming and multi agent-based simulation. Recent studies have proposed methods for efficient optimization of SAV routing, etc. [18]–[21], but they have not been applied to MOOPs.

II. Literature review

Application of automated, connected, and/or ridesharing vehicles to intelligent transportation systems has been extensively studied in the recent literature. For comprehensive and detailed reviews, readers may refer to [2], [22]–[24].

SAV systems may change operational aspects of transportation systems (e.g., traffic volume, travel time) first. Then, the change of operation may influences strategic aspects of transportation and urban systems (e.g., road network, land use). Therefore, operational and strategic aspects of SAV systems are interrelated. In this section, existing studies on operational and strategic aspects of SAV systems that are closely related to this study are reviewed, and the originality of this study is highlighted.

A. Operational aspects of SAV systems

VRPPDTW problems and their extensions for SAV systems have been extensively studied in the literature. SAV systems may able to do more efficient routing and passenger pickup and drop-off because of the automated driving and cooperation among SAVs. One of the most popular approaches is mixed integer linear programming (MILP) on time-expanded networks [8], [24], in which passengers and SAVs are dynamically matched. Furthermore, ridesharing can be directly incorporated to the MILP framework [9], [10], [24], [25]. The objective functions of such MILPs are commonly a system-wide performance index such as total travel time and vehicle distance traveled; this implicitly or explicitly assumes that the SAV systems are operated by a central operator. Decentralized optimization has also been studied for computational efficiency or robustness against failures [24].

Vehicle routing problems have been also modeled as a continuous (i.e, macroscopic) DTA problems, in which vehicles and travelers are described by continuous variables such as traffic volume [18], [19]. The advantage of this approach is that it is much more computationally efficient than MILP-based models while maintaining accurate representation of traffic dynamics such as congestion. The disadvantage is that it may not derive itinerary of individual vehicles and travelers; however, this limitation is common in most of conventional traffic assignment problems [26].

Ridesharing problems have been also modeled by using the matching theory, which describes the matching process by users' economical behavior [12], [27], [28]. The advantage of this approach is that it describes users' behavior more appropriate in terms of their profit-seeking attitudes.

Another popular approach is heuristic algorithms based on microscopic information such as trip requests [1], [11], [15]. Its advantage is that it is computationally efficient and can be flexibly applied to on-demand services and other complicated or practical situations.

B. Strategic aspects of SAV systems

In the long-term, SAV systems may influences some important factors in entire urban transportation systems. Therefore, it might be desirable to design SAV systems that would realize efficient urban transportation systems [2], [23], [29].

The fleet sizing problem is important for SAV systems[2]. Because of the efficient routing and ridesharing, the number of required SAVs to serve a fixed travel demand may be substantially smaller than that of conventional vehicles. To solve this problem, various approaches have been proposed. First, well-defined mathematical optimization problems is used to directly find the minimum fleet size [5], [30]. This approach is often incorporated with the MILP for VRPPDTW and/or ridesharing problems. Second, application of the graph theory is utilized for computational efficiency [3], [4]. Third, microscopic agent-based simulation is often employed [1] for its flexibility.

The road network design problem is also considerable in the long-term, because traffic demand changes substantially due to SAV systems. Especially, the autonomous vehicle lane deployment problem may be practically important in the near future, because it only requires relatively minor modification to the current infrastructure. The most common approach is application of traffic assignment methods [6], [31] that quantify relation between system performance and road network design of transportation systems. In this approach, continuous and static models have been utilized [6], [31].

Parking space allocation may change significantly due to SAV systems. This is because SAVs can drop off passengers at their convenient locations (e.g., homes, offices, locations with usually high land price) and then park at distant location with low land price, and this phenomenon may change land use pattern of a city [22]. To solve this problem, approaches

[2]This may be considered as an operational problem. However, we categorize this as strategic because its time-scale is substantially longer than other operational problems such as routing; time-scale for routing problems may be minutes or less, but that for fleet sizing problems may be months or more.

similar to that for the road network design problem have been used, such as continuous traffic assignment [32] and microscopic agent-based simulation [33]. More macroscopic approaches based on spatial economics have also been used [7], [34].

Apart from the above specific problems, trade-off relation between performance indexes of SAV systems have been noted. As mentioned in Section I, design of SAV systems may vary significantly depending on the aim of the systems, and user-side cost and system-side cost may vary depending on the design [13]–[15]. However, to the authors' knowledge, there are only a few studies that directly investigated this issue in quantitative approaches. [15] investigated trade-off relations between operation efficiency and quality of service by microscopic traffic simulation. [14] also investigated similar trade-off relations by using an MILP-based model. [16] proposed a Bayesian optimization framework integrated with an agent-based simulation in order to simultaneously solve the operational and strategic problems while considering the conflict between public and private sectors.

The MOOP is a powerful and rigorous methodology to investigate the aforementioned trade-off relations [17]. It derives set of all the Pareto efficient states that simultaneously optimize various objective functions. However, to the authors' knowledge, no studies have applied the MOOP to SAV systems. As a relevant study, [35] formulated an MOOP of shared electric vehicles in order to investigate trade-off relation between travel efficiency, charging efficiency, and cost.

C. Originality of this study

Based on the review, the original contribution of this study can be summarized as follows:

- Strategic decision making of SAV systems with explicit and endogenous consideration to dynamic operational aspects.
- A tractable multi-objective optimization for SAV systems.

The primary aim of the proposed model is to help strategic decision making of SAV systems, such as fleet sizing, road network design, and parking space allocation. Given the importance of interrelationship between strategic aspects and operational ones of SAV systems, the model also considers dynamic operation of SAVs, such as routing, passenger pickup/drop-off, and ridesharing in a macroscopic manner.

By employing a continuous DTA modeling approach, the model is formulated as a multi-objective linear optimization problem that is highly tractable. This tractability can be valuable given the fact that general MOOPs can be very difficult to solve. Furthermore, thanks to the tractability, mathematically proof on an important feature of the model is rigorously provided (Section III-E).

III. Formulation

A. Overview

The key assumptions employed by the proposed model are as follows. An *SAV* is defined as a driver-less vehicle that is shared by a society and may be used by any traveler in the society. The behavior of SAV traffic is described using a DTA model, where each link has traffic capacity and each node has storage capacity. *Ridesharing* is defined as an activity where an SAV carries two or more travelers simultaneously. An SAV may or may not be used as a ridesharing transportation mode. A *traveler* is defined as a person who has a specific origin, destination, earliest departure time from origin, and latest arrival time to destination. A time-dependent origin-destination (OD) matrix, which is aggregation of information of all travelers, is given.[3] A traveler is able to move only if s/he rides an SAV; otherwise, s/he must wait at a node. The society or the operator of SAV system determines a system optimal itinerary (i.e., routes), and all SAVs and travelers follow it.[4] The objective function for the system optimal may vary depending on the society's goal or the operator's type (e.g., public authority or private company); the proposed MOOP considers various types of such objective functions simultaneously.

Based on the above assumptions, we propose an MOOP that jointly minimizes the

- total travel time of travelers,
- total distance traveled by SAVs,
- total number of SAVs, and
- total infrastructure construction cost,

in an urban area in which the mean of transportation is an SAV system. The decision variables are such as

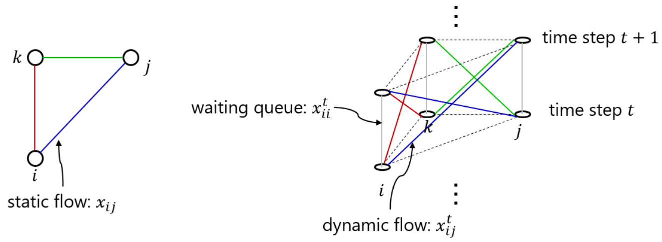
- SAV's routing (including empty vehicles and occupied vehicles),
- passenger's assignment,
- total number of SAVs, and
- link capacity and storage capacity of a location.

B. Problem

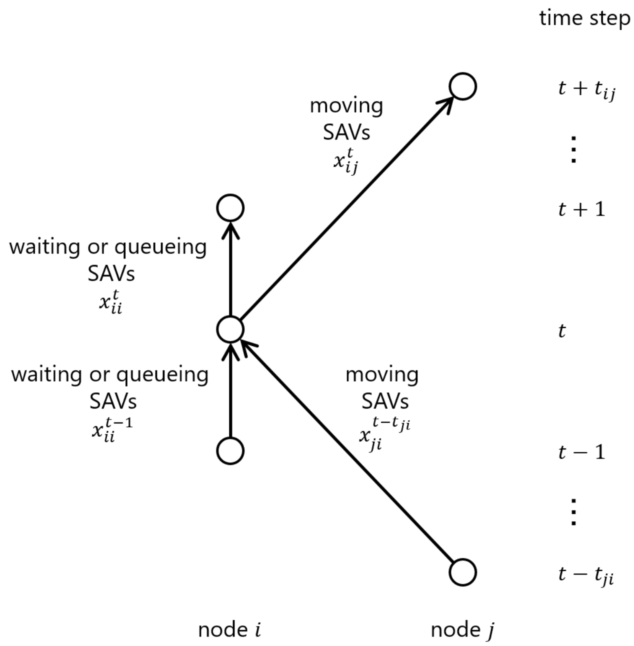
The basic structure of the problem is as follows. We adopt the concept of the maximal flow problem with a time-expanded network [36]. A road network is modeled as a time-expanded network shown in Fig. 1a, and time-dependent movements of travelers and SAVs are modeled as flows on the time-expanded network. Let x^t_{ij} be the total number of SAVs that travel from node i to j on time step t (Fig. 1b), and $y^{k,t}_{s,ij}$ be the total number of travelers with destination s and departure time k that travel from node i to j on time step t (Fig. 1c). As travelers need to ride SAVs to travel, the number of moving travelers on a certain link on a certain time step must be smaller than or equal to the number of moving SAVs times the passenger capacity of SAVs on the corresponding link and time step. This can be expressed as condition $\sum_{s,k} y^{k,t}_{s,ij} \leq \rho x^t_{ij}$, where ρ is a given passenger capacity of an SAV. Similarly, SAV flow must be smaller than or equal to the link capacity; thus, condition $x^t_{ij} \leq \mu_{ij}$ must be satisfied, where μ_{ij} is the link capacity. If SAV flow reaches its capacity (i.e., $x^t_{ij} = \mu_{ij}$), then the remaining SAVs will form a waiting queue on the node (i.e., $x^t_{ii} > 0$). Furthermore, the conservation law at a node must be satisfied; the sum of incoming flows to a node must be equal to that of outgoing flows, as depicted in Figs. 1b and 1c.

[3]This demand information can also be considered as a hypothetical demand that aggregates average on-demand request.

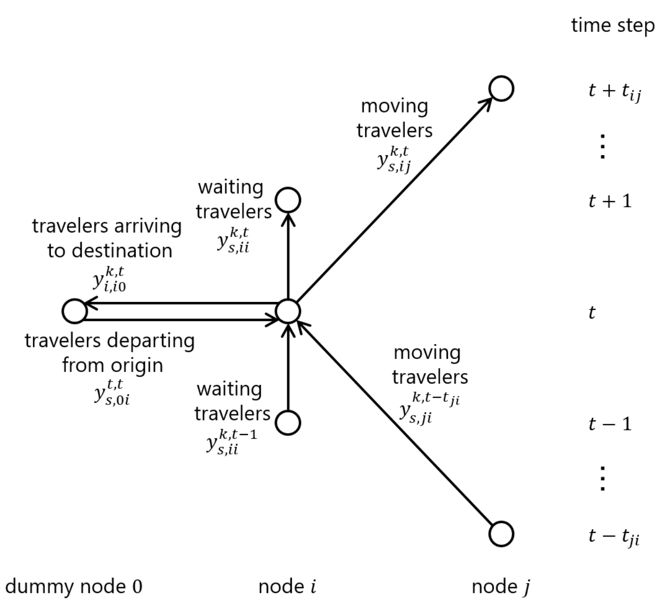
[4]This can be considered as reasonable as the SAV system is the only travel mode. We will discuss later how this assumption can be relaxed.



(a) Standard network (left) and time-expanded network for dynamic flow/congestion representation (right)



(b) SAV flow conservation at node i on time step t in time-expanded network



(c) Traveler flow conservation at node i on time step t in time-expanded network

Fig. 1: Dynamic traffic assignment using time-expanded network.

Our objective is to find the most efficient $y_{s,ij}^{k,t}$ and x_{ij}^t and other decision variables under proper constraints including the passenger and traffic capacity constraints.

As a result, the proposed social optimal SAV planning problem is formulated as follows:

$$\text{[SOSAV]} \qquad \min\,(T,\ D,\ N,\ C) \tag{1}$$

subject to

$$\sum_{ij,s,t,k} t_{ij} y_{s,ij}^{k,t} = T \tag{2}$$

$$\sum_{ij,i\neq j} d_{ij} x_{ij}^t = D \tag{3}$$

$$\sum_i x_{0i}^0 = N \tag{4}$$

$$\sum_{ij} c_{ij}(\mu_{ij} - \mu_{ij}^{\min}) + \sum_i c_i(\kappa_i - \kappa_i^{\min}) = C \tag{5}$$

$$\sum_j x_{ji}^{t-t_{ji}} - \sum_j x_{ij}^t = 0 \qquad \forall i, t \in (0, t_{\max}) \tag{6}$$

$$\sum_j y_{s,ji}^{k,t-t_{ji}} - \sum_j y_{s,ij}^{k,t} + y_{s,0i}^{k,t} - y_{s,i0}^{k,t} = 0 \qquad \forall i, s, k, t \in T_k \tag{7}$$

$$\sum_{s,k} y_{s,ij}^{k,t} \leq \rho x_{ij}^t \qquad \forall ij, i \neq j, t \tag{8}$$

$$x_{ij}^t \leq \mu_{ij} \qquad \forall ij, i \neq j, t \tag{9}$$

$$x_{ii}^t \leq \kappa_i \qquad \forall i, t \tag{10}$$

$$y_{s,0r}^{k,k} = M_{rs}^k \qquad \forall rs, k \tag{11}$$

$$\sum_{t\in T_k} y_{s,s0}^{k,t} = \sum_r M_{rs}^k \qquad \forall s, k \tag{12}$$

$$x_{ij}^t \geq 0 \qquad \forall ij, t \tag{13}$$

$$y_{s,ij}^{k,t} \geq 0 \qquad \forall ij, s, k, t \in T_k \tag{14}$$

$$x_{0i}^0 \geq 0 \qquad \forall i \tag{15}$$

$$y_{s,s0}^{k,t} \geq 0 \qquad \forall s, t, k \in T_k \tag{16}$$

$$\mu_{ij}^{\min} \leq \mu_{ij} \leq \mu_{ij}^{\max} \qquad \forall ij \tag{17}$$

$$\kappa_i^{\min} \leq \kappa_i \leq \kappa_i^{\max} \qquad \forall i \tag{18}$$

where the notation is summarized in Table I.

The meaning of each constraint is as follows. Eq. (2) is the definition of the total travel time, Eq. (3) is the definition of the total distance traveled by SAVs, Eq. (4) is the definition of the total number of SAVs, Eq. (5) is the definition of the total infrastructure construction cost, Eq. (6) is the conservation of SAVs at a node, Eq. (7) is the conservation of travelers at a node, Eq. (8) is the passenger capacity of an SAV, Eq. (9) is the traffic capacity of a link, Eq. (10) is the queue length capacity of a node, Eq. (11) is the traveler departure demand at an origin node, and Eq. (12) is the traveler arrival demand at a destination node.

The decision variables are x_{ij}^t (corresponding to VRPPDTW with ridesharing), $y_{s,ij}^{k,t}$ (VRPPDTW with ridesharing), μ_{ij} (link construction or SAV lane deployment problem), κ_i (parking space allocation problem) for all i, j, s, k, and t, N (fleet sizing problem), T, D, and C. Note that all of these

TABLE I: Mathematical notation

Symbol	Meaning
x_{ij}^t	flow of SAVs that start traveling link ij on time step t
$y_{s,ij}^{k,t}$	flow of travelers who start traveling link ij on time step t with origin node s and departure time step k
T	total travel time of travelers (including waiting time on nodes)
D	total distance traveled by SAVs
N	total number of SAVs
C	total cost of infrastructure construction
t_{ij}	free-flow travel time of link ij if $i \neq j$
t_{ii}	waiting time at node i for one time step (i.e., equal to the time step width by the definition)
d_{ij}	length of link ij
c_{ij}	unit cost of expanding traffic capacity of link ij
c_i	unit cost of expanding storage capacity of node i
ρ	passenger capacity of an SAV
μ_{ij}	traffic capacity of link ij
κ_i	storage capacity of node i
$\mu_{ij}^{\max}$	maximum allowable value of μ_{ij}
$\kappa_i^{\max}$	maximum allowable value of κ_i
$\mu_{ij}^{\min}$	minimum allowable value of μ_{ij}
$\kappa_i^{\min}$	minimum allowable value of κ_i
M_{rs}^k	time-dependent demand of travelers with origin r, destination s, and departure time step k
T_k	travel time window for travelers with departure time step k
$t_{\max}$	final time step

variables have a linear relationship in the problem. Thus, this is a linear programming problem. The computational complexity is polynomial of the number of links and time steps.

The problem can be categorized as a dynamic system optimal assignment that jointly minimizes T, D, N, and C based on a point-queue DTA model with a limited queue length (i.e., there may be queue spillback) and vehicle holding at nodes. These principles are similar to those of Kinematic Wave (KW) model [37], [38], which is the standard dynamic traffic flow model in the literature. In fact, it can be considered as a significant modification of the DTA method with the vehicle holding technique (which is almost equivalent to KW model but slightly violates the first-in first-out principle) proposed by [39] and the DTA-based optimal evacuation problems proposed by [40] and [41]. In order to construct a proper time-expanded network, the time discretization width should be equal to the space discretization width (which is usually equal to the link length) divided by the free-flow speed divided by an arbitrary positive integer.

C. Traffic dynamics features

In this subsection, we describe how the proposed model considers many important elements in dynamical modeling of SAV systems, namely,

- traffic congestion,
- empty vehicles' travel and detour due to ridesharing, and
- waiting time of passengers.

Traffic congestion is explicitly considered by the model. As previously mentioned, the traffic model is equivalent to a point-queue model with limited queue length, in which each link has traffic capacity μ_{ij}, and SAVs that cannot go through a link need to wait on a node. The waiting SAVs on a node form a waiting queue with length x_{ii}^t.

Travel of empty (i.e., passenger-less) vehicles is explicitly considered by the model. As the model utilizes the conservation law (6), SAVs do not appear, disappear, or warp during the operation. Therefore, if a drop-off location for an SAV's trip and its next pick-up location were different, the SAV would need to travel to the pick-up location via the network. The number of empty vehicles that travel link ij on time step t can be estimated as $x_{ij}^t - \sum_{s,k} y_{s,ij}^{k,t}/\rho$. As one of the objectives of this problem is minimization of the total distance traveled $D = \sum_{ij,i\neq j} d_{ij} x_{ij}^t$, the model tries to minimize such trips made by such empty SAVs as much as possible. This feature will be useful to tackle the important issue on congestion caused by travel of empty vehicles and parking space allocation problems, which have been observed in actual operations of transport network companies [42].

Detours due to ridesharing are also explicitly considered by the model, owing to the conservation laws (6) and (7). The mechanism is similar to that of travel of empty vehicles.

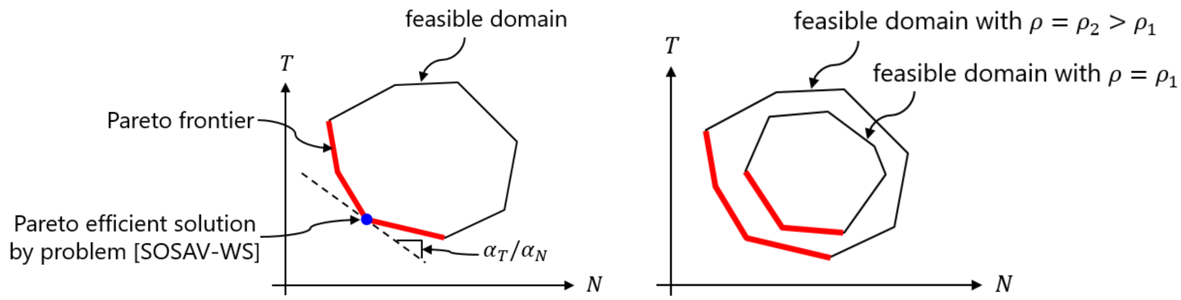
The waiting time of passengers is considered by the model. As SAVs have limited passenger capacity and links have limited traffic capacity, the number of movable travelers is always bounded. Travelers who cannot move need to wait on a node.

The above features are numerically demonstrated in Appendix A.

D. Solution method

An MOOP is "solved" when its *Pareto frontier*—a set of all of the *Pareto efficient solutions*—is derived [17]. The definition of a Pareto efficient solution of [SOSAV] is a vector (T, D, N, C) where any of the objective function values cannot be decreased without increasing the other(s). As [SOSAV] is a minimization problem, its Pareto frontier is the lower envelope of its feasible solution domain (see Fig. 2a). A decision-maker would select one of the solutions from the Pareto frontier by considering the society's policies and trade-off relations among the objective functions.

The proposed problem can be efficiently solved owing to its linearity. For example, standard methods such as the weighted sum method and multi-objective simplex method can



(a) Pareto frontier

(b) Ridesharing always improves the Pareto frontier (Theorem 1)

Fig. 2: Pareto frontier in an MOOP. Note that only the two-dimensional objective domain (T, N) is shown in this figure for conceptual illustration purpose, although the actual objective domain is four-dimensional (T, D, N, C).

be adopted [17]. This is a notable advantage of the proposed problem, because general MOOPs tend to be difficult to solve.

In this study, the weighted sum method is employed. The method approximates a Pareto frontier by iteratively computing a Pareto efficient solution by solving following single-objective linear optimization:

$$\text{[SOSAV-WS]} \quad \min \alpha_T T + \alpha_D D + \alpha_N N + \alpha_C C \tag{19}$$

subject to Eq. (2)–(18), where each α_T, α_D, α_N, and α_C are given non-negative constants representing the priority of the corresponding objective function. Because of the linearity of [SOSAV], it is guaranteed that a solution of [SOSAV-WS] is always a Pareto efficient solution of [SOSAV], and all of the Pareto efficient solutions of [SOSAV] are solutions of [SOSAV-WS] with appropriate α. Thus, a Pareto frontier of [SOSAV] can be approximated as a set of solutions with [SOSAV-WS] with different α.

E. Qualitative properties

Problem [SOSAV] has useful policy implication on ridesharing. It is mathematically guaranteed that the optimal values of the total travel time T, total distance traveled D, number of SAVs N, and construction cost C of [SOSAV] are simultaneously and monotonically non-increasing by increasing the passenger capacity ρ. Thus, ridesharing in [SOSAV] is always beneficial to average travelers as well as vehicle operators, road authorities, and the environment in the proposed model. This property is stated as a mathematical theorem as follows.

Theorem 1. *For all $\rho_2 > \rho_1 > 0$ and for all Pareto efficient solutions in [SOSAV] with $\rho = \rho_1$, there exists more weakly efficient solutions in [SOSAV] with $\rho = \rho_2$.*

Proof. In [SOSAV], parameter ρ appears in Eq. (8) only. According to Eqs. (8), (13), and (14), the feasible domain of $y^{k,t}_{s,ij}$ and x^t_{ij} monotonically expands (i.e., the upperbound increases weakly monotonically and the lowerbound decreases weakly monotonically) as ρ increases. Hence, the feasible domain of T, D, N, and C also monotonically expands as ρ increases. In other words, [SOSAV] relaxes as ρ increases. Consequently, given the definition of a Pareto frontier (i.e., the lower envelope of a feasible domain) and the fact that [SOSAV] is a linear minimization problem, the theorem holds true. □

See Fig. 2b for conceptual illustration of this theorem. If constraint (8) is active, T, D, N, and C can be simultaneously and monotonically decreased by increasing ρ. Note that the constraint (8) is very likely to be active under realistic data, because it represents the passenger capacity constraint.

It is worth noting that, according to Theorem 1, the introduction of ridesharing can reduce the average travel time of travelers. In the literature, the reduction of vehicle distance traveled and fleet size is highlighted as a benefit of ridesharing; however, the travel time of travelers tends to be increased owing to the detours and waiting time caused by ridesharing. Theorem 1 implies that the travel time of travelers can also be reduced by ridesharing if SAVs are properly operated.

F. Limitations

The proposed model has several limitations for the sake of simplicity and tractability. In this subsection, the limitations and their implications are discussed. Note that some of the limitations also exist in many studies in the literature.

The demand in the proposed model is deterministically known. In the real-world, the demand is unknown or uncertain. Thus, the model overestimates the efficiency of SAV systems. In order to overcome this issue, the robust optimization approach would be considerable.

The model only computes aggregated link flows; therefore, path flows and travel routes of individual travelers cannot be identified uniquely. Furthermore, it may be impossible to decompose the flows into discrete travelers.

Although above two issues are clear limitations of the proposed model, similar features are often assumed to model SAVs, automated vehicle operations, or ridesharing services in the literature [6], [18], [19], [43], [44]. Therefore, the proposed model could be still useful for designing SAV systems in the strategic planning phases; as discussed in Section I, this role is essential for urban transportation planning for example.

The model ignores the time and cost associated with passengers' getting in/off SAVs. Therefore, the number of passengers switching SAVs may be too large in the solution. However, as we will see in Appendix A, the number was actually small.

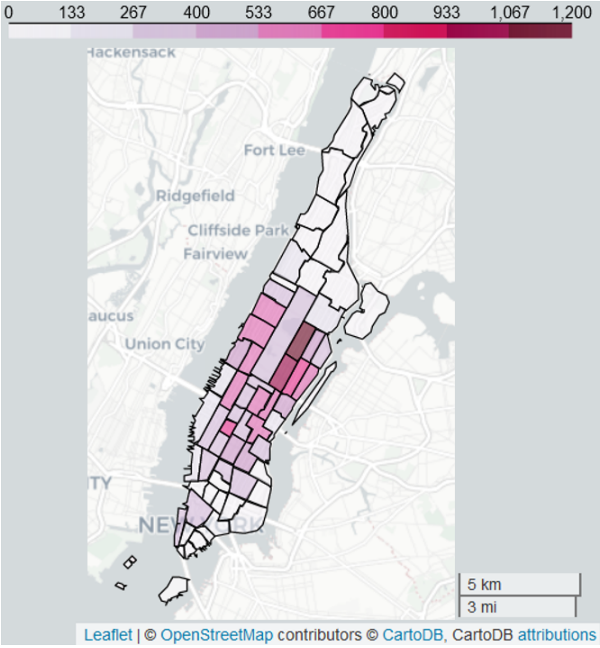


Fig. 3: Passenger demand distribution.

Perhaps solutions with too many switching are not efficient because it would indirectly increase travel time and distance traveled by SAVs.

SAV systems are assumed to be only available transportation mode. This is clearly a strong assumption. This can be relaxed by introducing a travel mode choice model as an upper level problem of the current model.

As mentioned, the model requires special spatial-temporal discretization. This is not necessarily exactly applicable to general networks. Hence, approximation of network structure or size may be required.

IV. Numerical experiment

To investigate the quantitative behaviors of the proposed model under somewhat realistic conditions, a numerical experiment with actual travel data from New York City (NYC) was conducted.

A. Scenario

The passenger demand was generated from the NYC taxi data [45]. The taxi data include the origin zone, destination zone, departure time from origin, and travel time for each individual passenger's trip. The travel records of taxis from 8:00 to 9:00 on 2019-04-01 (Monday) in Manhattan were extracted. We assumed that these travel records were equivalent to travel requests by SAV users in this area. Fig. 3 shows the spatial distribution of passenger demand in the area. The total number of passengers was 17,998. Then, the travel requests were aggregated to the time-dependent OD matrix M_{rs}^k with a 5 min time discretization width and a 30 min departure time aggregation width.

Regarding ridesharing, three cases were considered: no-ridesharing ($\rho = 1$), two-person ridesharing ($\rho = 2$), and five-person ridesharing ($\rho = 5$).

The road network was generated as follows. Because the passenger demand data are zone-based (see Fig. 3), we considered each zone as a node in the network, and each neighboring zone were connected by a link. The number of nodes was 67. The free-flow link travel time t_{ij} was assumed as 5 min, which is similar to the trip records for many links. The approximate land value for each zone was obtained from [46], and the values of c_{ij} and c_i were determined as proportional to the land value. Note that the unit of the land value was artificial (i.e., not an actual currency) in this study.

The other model parameters were set as follows: $\mu_{\min} = 4$, $\mu_{\max} = 40$, $\kappa_{\min} = 4$, $\kappa_{\max} = 40$, and the maximum allowable travel time was 30 min.

B. Results

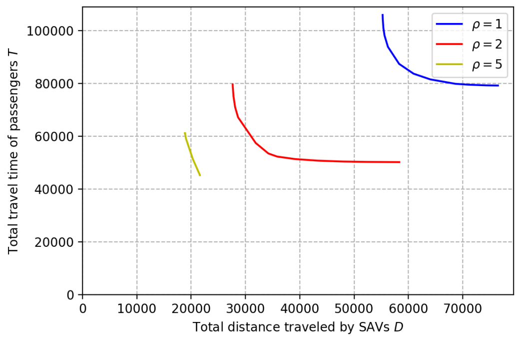
The solution (i.e., Pareto frontier) of [SOSAV] for each ρ case was obtained by iteratively solving [SOSAV-WS] using the interior point method implemented by Gurobi [47]. An instance of [SOSAV-WS] was solved in about 2–4 min using 6 threads of a 3.79 GHz CPU and 3 GB of RAM, which can be considered as reasonably efficient.

The Pareto frontier of [SOSAV] is a four-dimensional object, which hard to be illustrated directly on this paper. Therefore, in what follows, several aspects of the Pareto frontiers are described in order to understand the features of the model's solution.

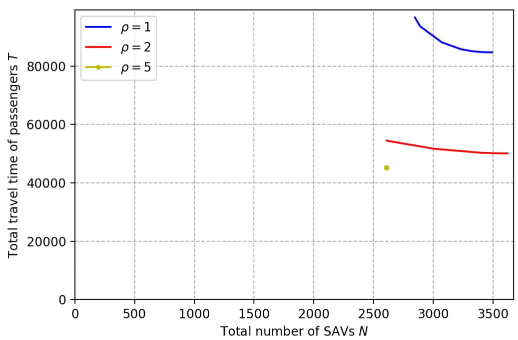
Pareto frontiers in two-dimensional domains are illustrated in Fig. 4. In these plots, the relation between two objective values (i.e., T and one of the others) is shown where the rest of the objective values are fixed to certain values (i.e., $D = 60000$, $N = 3500$, $C = 100000$). According to the figure, the trade-off relation between the objectives were clearly found. For example, in the no-ridesharing case ($\rho = 1$) in Fig. 4c, if minimization of passenger's travel time T is prioritized, the infrastructure cost C will become significantly high. On the other hand, if minimization of the infrastructure cost C is prioritized, C can be reduced by half while T will be increased by 1.5 times. Similar tendencies were also confirmed in Figs. 4a and 4b. An SAV system planner could choose these two extreme cases as well as various moderate cases between them on the Pareto frontier, depending on the political/strategic goal of the SAV system and society.

By comparing the no-ridesharing cases ($\rho = 1$) to the two-person ridesharing cases ($\rho = 2$) in Fig. 4, the efficiency of ridesharing, which is theoretically guaranteed by Theorem 1, was evident. For example, according to Fig. 4c, the total travel time of passengers T of $\rho = 2$ cases is almost half of that of $\rho = 1$ cases when the same infrastructure cost C is given. Similarly, C of $\rho = 2$ cases is less than half (sometimes a third or fourth) of that of $\rho = 1$ cases when the same T is given. Similar tendencies were also confirmed in five-person ridesharing cases and Figs. 4a and 4b. This demonstrates that introduction of ridesharing is generally beneficial to various members of the society in this model.

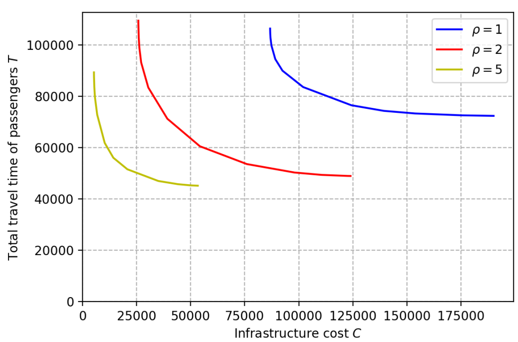
Ranges of objective values in Pareto efficient solutions are shown in Fig. 5. According to the figure, the objective values can take a wide range of values; for example, if one of the



(a) (T, D) domain where N and C are fixed.



(b) (T, N) domain where D and C are fixed.



(c) (T, C) domain where D and N are fixed.

Fig. 4: Pareto frontiers.

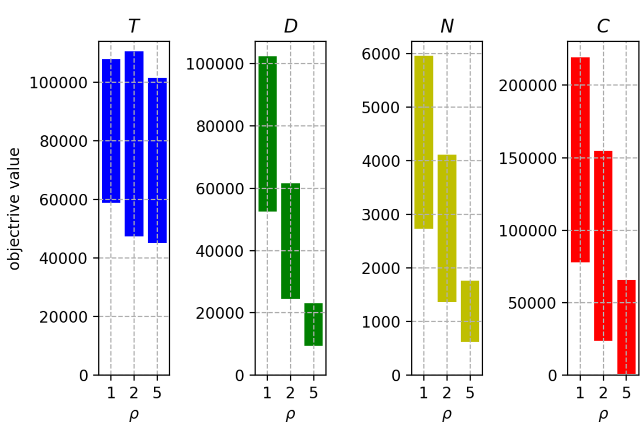


Fig. 5: Ranges of objective values in Pareto efficient solutions without ridesharing ($\rho = 1$) or with ridesharing ($\rho = 2$ or 5).

objectives was prioritized, its value could be half of that of cases with no priority on the objective. Furthermore, the introduction of ridesharing can greatly reduce the objective values. For example, the average number of passenger trips served by an SAV is an important performance index of SAV systems. According to the range of N in Fig. 5, the number was 3.0–6.7 passengers/SAV/h for the no-ridesharing case, 4.4–13 passengers/SAV/h for the two-person ridesharing case, and 10–30 passengers/SAV/h for the five-person ridesharing case (the total number of the passengers was 17,998). These results again confirmed that an SAV system planner needs to carefully design an SAV system through MOOP.

Figs. 6–8 shows the spatial distributions of the model's variables in several Pareto efficient solutions. Each upper row represents cases without ridesharing ($\rho = 1$), and each lower row represents cases with ridesharing ($\rho = 2$). Each column represents cases where specific objective function was prioritized. The "priority on T" means that the Pareto efficient solution was obtained by setting $\alpha_T = 100$, $\alpha_D = 1$, $\alpha_N = 1$, and $\alpha_C = 1$, and so on. For example, the upper right plot in Fig. 6 represents spatial distribution of SAV flow when the SAV system was designed with priority on C without ridesharing ($\rho = 1$).

Fig. 6 shows the distribution of $\sum_{t,j} x_{ij}^t$ where $i \neq j$, by which characteristics of SAV flow can be confirmed. When the minimization of T was prioritized (left column), high SAV flow would be observed to enable quick transportation of travelers. This high flow included travel of empty vehicles. Contrary, when the other objectives were prioritized, the SAV flow would be significantly reduced. The introduction of ridesharing ($\rho = 2$, lower row) also reduced the SAV flow. The distribution of SAV flow was similar to the demand distribution in Fig. 3.

Fig. 7 shows the distribution of $\sum_{t,s,k} y_{s,ii}^{k,t}$, by which characteristics of passenger's accumulated waiting time can be confirmed. When the minimization of T was prioritized, the waiting time was generally small. Contrary, when the other objectives were prioritized, the waiting time was increased, especially where large demand was present. Both of the no-ridesharing and ridesharing cases have similar waiting time distributions. This means that ridesharing does not necessarily decrease passenger's waiting time; this would be due to the time required for matching in ridesharing.

Fig. 8 shows the distribution of κ_i, by which characteristics of parking space can be confirmed. When minimization of T or D was prioritized, the parking spaces were maximized almost everywhere to enable quick response to traveler's demand or reduction of empty SAVs's running. Contrary, when the other objectives were prioritized, the parking spaces were placed only in locations with relatively low land value. Note that, in these two cases, the total parking space was not sufficient to park all SAVs simultaneously; therefore, many empty SAVs kept traveling on the road until they were assigned to new passengers. The introduction of ridesharing slightly reduced the parking space in all cases.

In Appendix A, traffic dynamics in a toy network are shown as a complementary analysis. According to the results, we can conclude that the proposed model properly represented

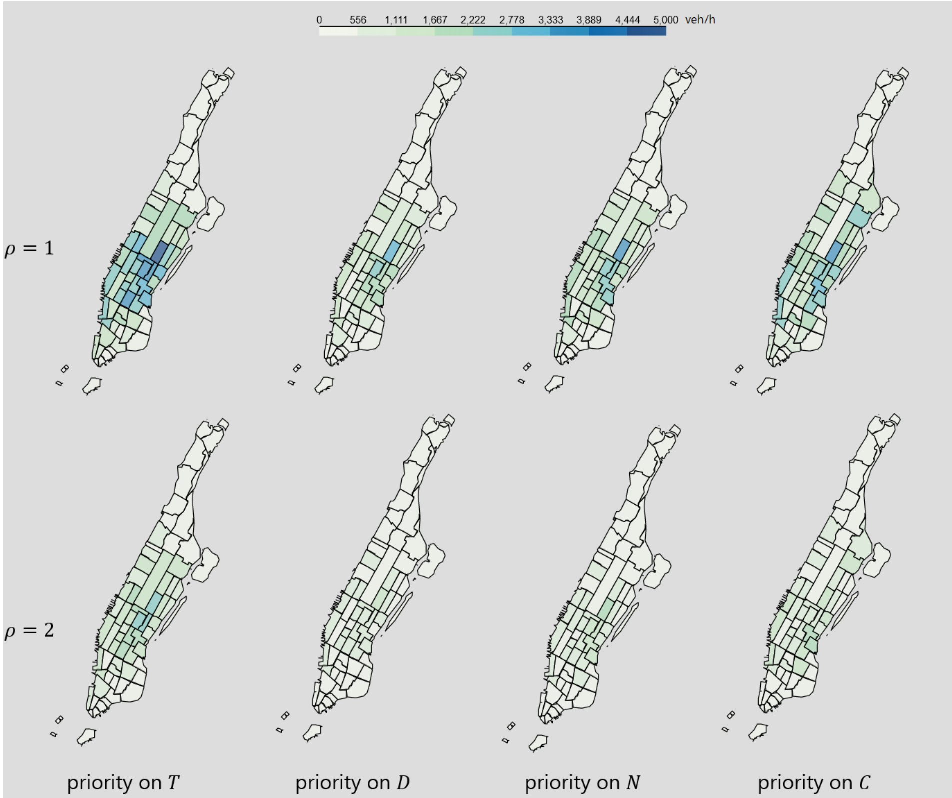


Fig. 6: Spatial distribution of SAV flow under various conditions.

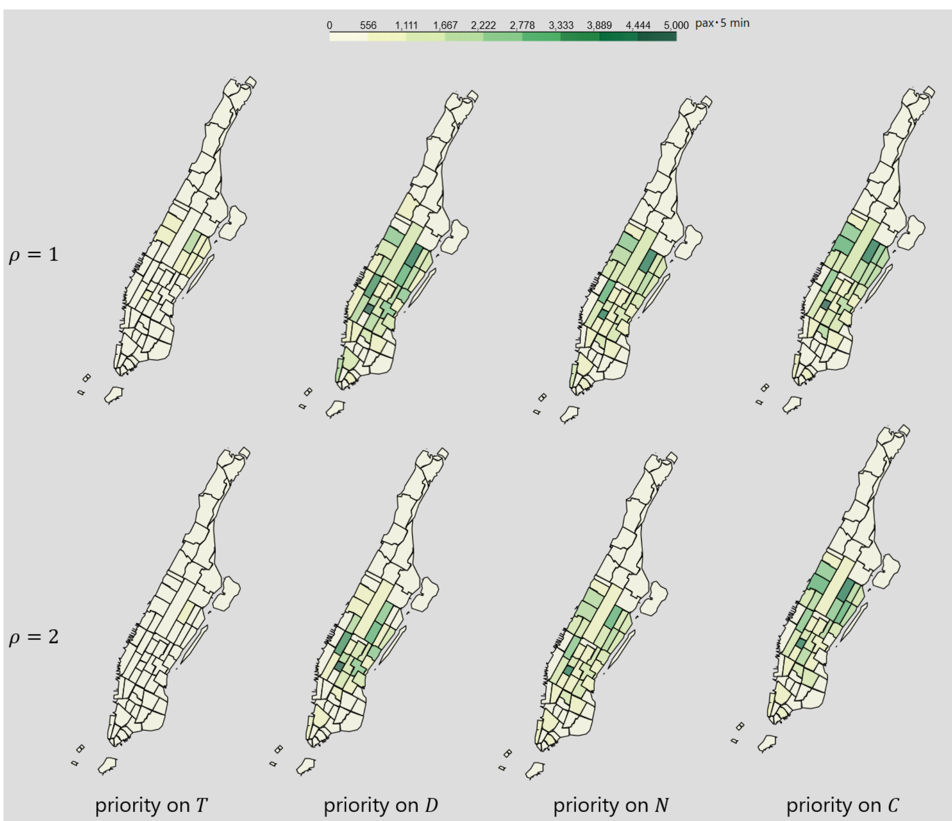


Fig. 7: Spatial distribution of passenger waiting time under various conditions.

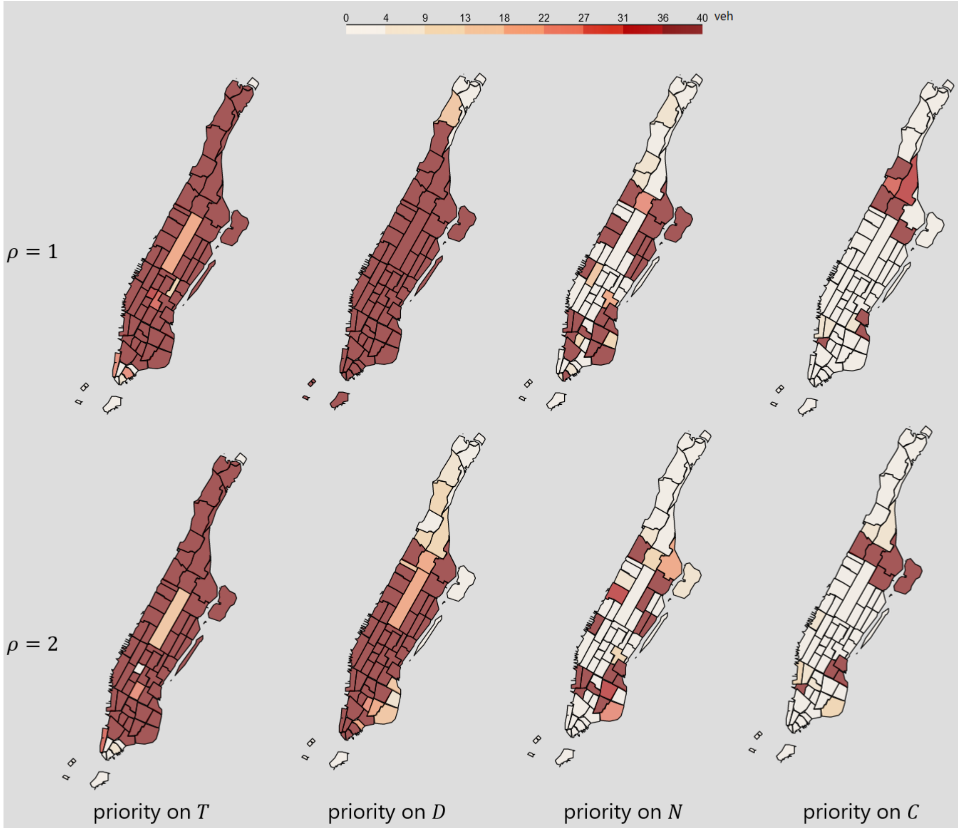


Fig. 8: Spatial distribution of parking slots under various conditions.

traffic dynamics (e.g., features mentioned in Section III-C) with ridesharing.

V. Conclusion

A multi-objective linear optimization problem that jointly optimizes aggregated variables on SAV's routing and passenger pickup/delivery (e.g, location, duration, and volume of them), traveler assignment and ridesharing, fleet sizing, road network design, and parking space allocation is proposed. Note that the model describes aggregated flow of SAVs and passengers only. Thus, the results might not be directly useful for operational decision making of SAV systems; instead, they can be useful for strategic decision making. It seeks to minimize various types of cost associated to each member of society, namely, total travel time of passengers, total distance traveled by SAVs, total number of SAVs, and infrastructure cost. Therefore, planners of SAV systems can select the best solution that reflects the society's political/strategic goal from a solution set obtained from the proposed problem. The proposed model is based on a DTA model (i.e., networked point queue models with limited queue length); thus, it explicitly considers dynamical features of SAV systems such as congestion propagation, empty vehicle routing, and passenger waiting time.

Mathematical analysis reveals that introduction of ridesharing simultaneously and weakly monotonically increases the utilities of average travelers, operators, and society in the proposed model. This feature as well as the mathematical tractability of the problem would be useful as a benchmark for SAV planning.

Numerical examples are presented to show quantitative behaviors of the model based on NYC taxi data. The results suggest that the proposed model behaves reasonably. It is noteworthy that features (i.e., SAV assignment, infrastructure design, operational performance) of a Pareto efficient solution could be completely different from those of other Pareto efficient solutions. This result implies that SAV systems need be carefully planned to ensure that the society's political/strategic goal is achieved; MOOPs such as the proposed problem would be useful for such a purpose.

The most important future work is to consider other travel modes such as public transit and private modes. This is because one of the assumptions of the current model (i.e., the SAV system is the only available travel mode) is somewhat restrictive. This extension can be achieved by introducing a travel mode choice model as an upper level problem of the current model.

Appendix A
Numerical example in a hypothetical one-dimensional city

In this appendix, the results of a small-scale experiment are described in order to illustrate detailed traffic dynamics in the model.

A. Scenario

Commuting traffic in a hypothetical one-dimensional city was generated. The number of nodes was 10, and the total number of travelers was 1000. Most of the travelers' origins were on the "left-side" of the city, and their destinations were on the "right-side". In these settings, traffic flow dynamics is easy to illustrate owing to its one-dimensional structure.

B. Results

The space–time diagrams of the travelers and SAV flows in some of the Pareto efficient solutions are shown in Fig. 9, which depicts the traffic dynamics in detail. In the figure, scenarios with and without ridesharing ($\rho = 1$ or 2) were solved and illustrated. In both scenarios, commuters were assumed to tend to travel from left to right following the same given OD distribution. In Fig. 9a, SAV flows on many links were saturated (i.e., the red colored flow in the right plot); thus, travelers needed to wait for long time at their origin nodes (i.e., thick vertical lines in the left plot). On the other hand, in Fig. 9b, the SAV flow was almost uncongested, and efficient traffic was realized because of ridesharing. This suggests that the proposed model properly represented traffic dynamics with ridesharing.

In the passenger space–time diagrams, most of the passengers traveled in straight lines and experienced no or few stops in the middle of travel. This implies that the passengers tend not to switch SAVs too often.

The Pareto frontiers of [SOSAV] with different ρ values are shown in Fig. 10. Similar to Fig. 4, we can easily confirm a trade-off relation between T and N and the positive effect of ridesharing. Furthermore, a scenario with fully private vehicles (i.e., no SAVs, no ridesharing), which was not mathematically feasible in the NYC case study, is also shown as a reference point. Compared with the fully private vehicles case, SAVs always reduced N; furthermore, ridesharing could reduce T. These can be considered as reasonable and desirable features of the SAV systems.

REFERENCES

[1] D. J. Fagnant and K. M. Kockelman, "Dynamic ride-sharing and optimal fleet sizing for a system of shared autonomous vehicles," in *Transportation Research Board 94th Annual Meeting*, 2015.

[2] S. Narayanan, E. Chaniotakis, and C. Antoniou, "Shared autonomous vehicle services: A comprehensive review," *Transportation Research Part C: Emerging Technologies*, vol. 111, pp. 255–293, 2020.

[3] P. Santi, G. Resta, M. Szell, S. Sobolevsky, S. H. Strogatz, and C. Ratti, "Quantifying the benefits of vehicle pooling with shareability networks," *Proceedings of the National Academy of Sciences*, vol. 111, no. 37, pp. 13 290–13 294, 2014.

[4] M. M. Vazifeh, P. Santi, G. Resta, S. H. Strogatz, and C. Ratti, "Addressing the minimum fleet problem in on-demand urban mobility," *Nature*, vol. 557, pp. 534–538, 2018.

[5] H. Zhang, C. J. Sheppard, T. E. Lipman, and S. J. Moura, "Joint fleet sizing and charging system planning for autonomous electric vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 11, pp. 4725–4738, 2019.

[6] Z. Chen, F. He, L. Zhang, and Y. Yin, "Optimal deployment of autonomous vehicle lanes with endogenous market penetration," *Transportation Research Part C: Emerging Technologies*, vol. 72, pp. 143–156, 2016.

[7] R. Zakharenko, "Self-driving cars will change cities," *Regional Science and Urban Economics*, vol. 61, pp. 26–37, 2016.

[8] M. Mahmoudi and X. Zhou, "Finding optimal solutions for vehicle routing problem with pickup and delivery services with time windows: A dynamic programming approach based on state-space-time network representations," *Transportation Research Part B: Methodological*, vol. 89, pp. 19–42, 2016.

[9] S. Aiko, R. Itabashi, T. Seo, T. Kusakabe, and Y. Asakura, "Social benefit of optimal ride-share transport with given travelers' activity patterns," *Transportation Research Procedia*, vol. 27, pp. 261–269, 2017.

[10] R. Regue, N. Masoud, and W. Recker, "Car2work: Shared mobility concept to connect commuters with workplaces," *Transportation Research Record*, vol. 2542, no. 1, pp. 102–110, 2016.

[11] M. W. Levin, K. M. Kockelman, S. D. Boyles, and T. Li, "A general framework for modeling shared autonomous vehicles with dynamic network-loading and dynamic ride-sharing application," *Computers, Environment and Urban Systems*, vol. 64, pp. 373–383, 2017.

[12] P. Thaithatkul, T. Seo, T. Kusakabe, and Y. Asakura, "Evolution of a dynamic ridesharing system based on rational behaviour of users," *International Journal of Sustainable Transportation*, vol. 13, no. 8, pp. 614–626, 2019.

[13] I. Docherty, G. Marsden, and J. Anable, "The governance of smart mobility," *Transportation Research Part A: Policy and Practice*, vol. 115, pp. 114–125, 2018.

[14] M. Hyland and H. S. Mahmassani, "Operational benefits and challenges of shared-ride automated mobility-on-demand services," *Transportation Research Part A: Policy and Practice*, vol. 134, pp. 251–270, 2020.

[15] C. Ruch, C. Lu, L. Sieber, and E. Frazzoli, "Quantifying the efficiency of ride sharing," *IEEE Transactions on Intelligent Transportation Systems*, 2020.

[16] F. Dandl, R. Engelhardt, M. Hyland, G. Tilg, K. Bogenberger, and H. S. Mahmassani, "Regulating mobility-on-demand services: Tri-level model and bayesian optimization solution approach," *Transportation Research Part C: Emerging Technologies*, vol. 125, p. 103075, 2021.

[17] M. Ehrgott, *Multicriteria Optimization*. Springer-Verlag Berlin Heidelberg, 2005.

[18] M. W. Levin, "Congestion-aware system optimal route choice for shared autonomous vehicles," *Transportation Research Part C: Emerging Technologies*, vol. 82, pp. 229–247, 2017.

[19] M. W. Levin, M. Odell, S. Samarasena, and A. Schwartz, "A linear program for optimal integration of shared autonomous vehicles with public transit," *Transportation Research Part C: Emerging Technologies*, vol. 109, pp. 267–288, 2019.

[20] A. Simonetto, J. Monteil, and C. Gambella, "Real-time city-scale ridesharing via linear assignment problems," *Transportation Research Part C: Emerging Technologies*, vol. 101, pp. 208–232, 2019.

[21] J. Liu, P. Mirchandani, and X. Zhou, "Integrated vehicle assignment and routing for system-optimal shared mobility planning with endogenous road congestion," *Transportation Research Part C: Emerging Technologies*, vol. 117, 2020.

[22] D. Milakis, B. Van Arem, and G. P. Van Wee, "Policy and society related implications of automated driving: a review of literature and directions for future research," *Journal of Intelligent Transportation Systems*, 2017.

[23] D. Milakis, N. Thomopoulos, and B. Van Wee, Eds., *Policy Implications of Autonomous Vehicles*. Academic Press, 2020.

[24] A. Tafreshian, N. Masoud, and Y. Yin, "Frontiers in service science: Ride matching for peer-to-peer ride sharing: A review and future directions," *Service Science*, vol. 12, no. 2-3, pp. 44–60, 2020.

[25] N. Agatz, A. Erera, M. Savelsbergh, and X. Wang, "Optimization for dynamic ride-sharing: A review," *European Journal of Operational Research*, vol. 223, no. 2, pp. 295–303, 2012.

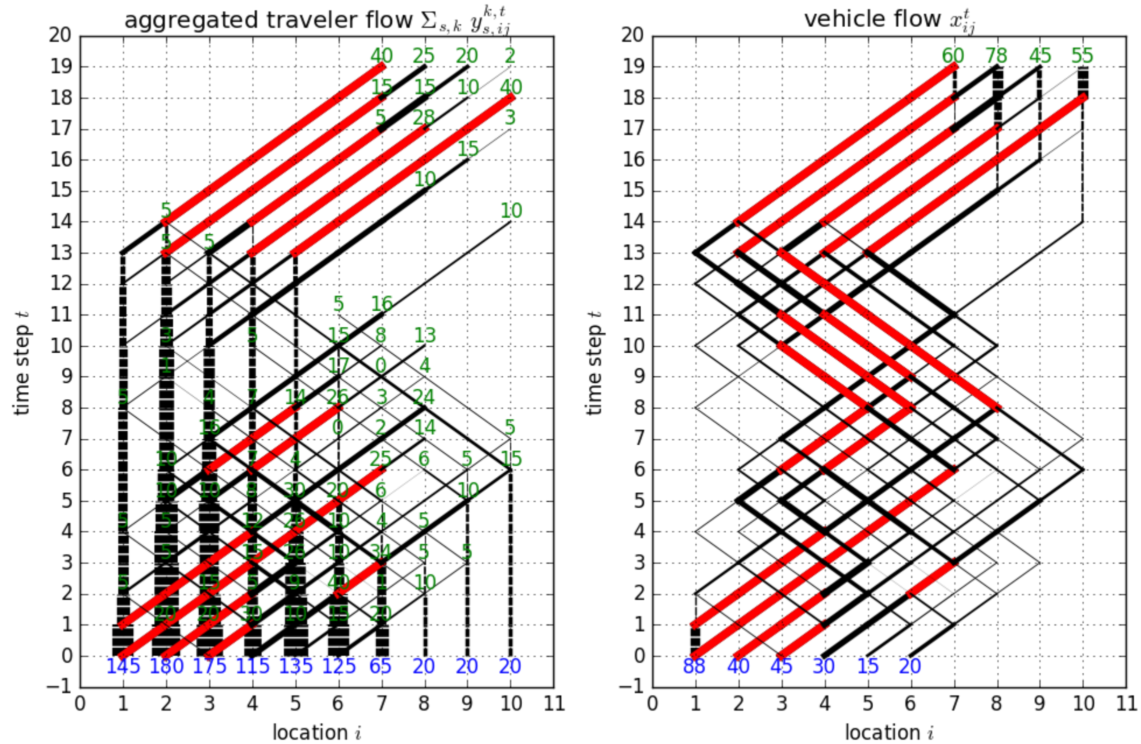
[26] M. P. Raadsen, M. C. Bliemer, and M. G. Bell, "Aggregation, disaggregation and decomposition methods in traffic assignment: historical perspectives and new trends," *Transportation Research Part B: Methodological*, vol. 139, pp. 199–223, 2020.

[27] X. Wang, N. Agatz, and A. Erera, "Stable matching for dynamic ride-sharing systems," *Transportation Science*, 2017.

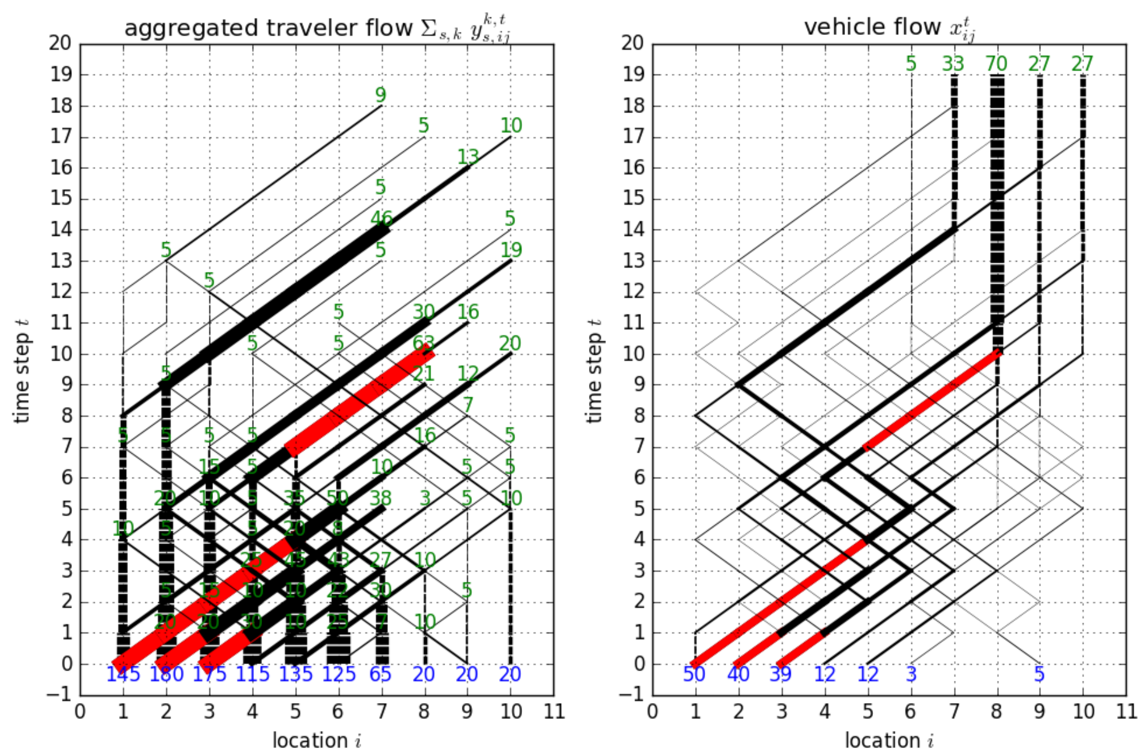
[28] H. Zhang and J. Zhao, "Mobility sharing as a preference matching problem," *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 7, pp. 2584–2592, 2018.

[29] D. J. Fagnant and K. M. Kockelman, "Preparing a nation for autonomous vehicles: opportunities, barriers and policy recommendations," *Transportation Research Part A: Policy and Practice*, vol. 77, pp. 167–181, 2015.

[30] M. Allahviranloo and J. Y. Chow, "A fractionally owned autonomous vehicle fleet sizing problem with time slot demand substitution effects," *Transportation Research Part C: Emerging Technologies*, vol. 98, pp. 37–53, 2019.



(a) SAV, no ridesharing ($\rho = 1$)



(b) SAV, 2 person ridesharing ($\rho = 2$)

Fig. 9: Space–time trajectories of travelers (left) and SAVs (right) in a hypothetical one-dimensional city. The width of each line represents the traffic volume. The horizontal lines in the traveler flow represent the travelers waiting at the nodes. The red lines represent saturated flow. The blue numbers represent inflow, and the green numbers represent outflow.

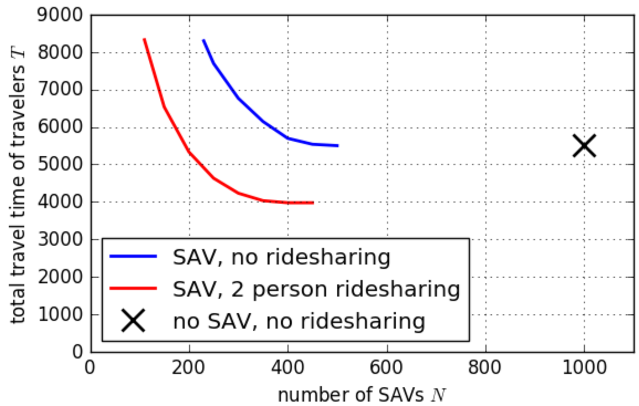
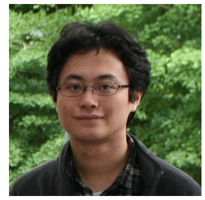


Fig. 10: Pareto frontiers on N and T in a hypothetical one-dimensional city.

[31] Z. Liu and Z. Song, "Strategic planning of dedicated autonomous vehicle lanes and autonomous vehicle/toll lanes in transportation networks," *Transportation Research Part C: Emerging Technologies*, vol. 106, pp. 381–403, 2019.
[32] M. W. Levin, E. Wong, B. Nault-Maurer, and A. Khani, "Parking infrastructure design for repositioning autonomous vehicles," *Transportation Research Part C: Emerging Technologies*, vol. 120, p. 102838, 2020.
[33] K. Winter, O. Cats, K. Martens, and B. van Arem, "Parking space for shared automated vehicles: How less can be more," *Transportation Research Part A: Policy and Practice*, vol. 143, pp. 61–77, 2021.
[34] J. Hawkins and K. N. Habib, "Integrated models of land use and transportation for the autonomous vehicle revolution," *Transport Reviews*, pp. 1–18, 2018.
[35] H. Miao, H. Jia, J. Li, and T. Z. Qiu, "Autonomous connected electric vehicle (ACEV)-based car-sharing system modeling and optimal planning: A unified two-stage multi-objective optimization methodology," *Energy*, vol. 169, pp. 797–818, 2019.
[36] L. R. Ford, Jr. and D. R. Fulkerson, "Constructing maximal dynamic flows from static flows," *Operations Research*, vol. 6, no. 3, pp. 419–433, 1958.
[37] M. J. Lighthill and G. B. Whitham, "On kinematic waves. II. a theory of traffic flow on long crowded roads," *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, vol. 229, no. 1178, pp. 317–345, 1955.
[38] P. I. Richards, "Shock waves on the highway," *Operations Research*, vol. 4, no. 1, pp. 42–51, 1956.
[39] A. K. Ziliaskopoulos, "A linear programming model for the single destination system optimum dynamic traffic assignment problem," *Transportation Science*, vol. 34, no. 1, pp. 37–49, 2000.
[40] M. Kuwahara, T. Ohata, and Y. Oishi, "Normative behaviour-based first-best analysis for disaster evacuation," *Transportmetrica B: Transport Dynamics*, 2017.
[41] Y. Hanawa, Y. Higashikawa, N. Kamiyama, N. Katoh, and A. Takizawa, "The mixed evacuation problem," *Journal of Combinatorial Optimization*, vol. 36, no. 4, pp. 1299–1314, 2018.
[42] B. Schaller, "The new automobility: Lyft, Uber and the future of American cities," Schaller Consulting, Tech. Rep., 2018.
[43] M. Ammous, S. Belakaria, S. Sorour, and A. Abdel-Rahim, "Optimal cloud-based routing with in-route charging of mobility-on-demand electric vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 20, no. 7, pp. 2510–2522, 2018.
[44] G. Guo and T. Xu, "Vehicle rebalancing with charging scheduling in one-way car-sharing systems," *IEEE Transactions on Intelligent Transportation Systems*, 2020.
[45] Taxi and Limousine Commission, "2019 yellow taxi trip data," 2020.
[46] B. Rankin, "Land value," 2006.
[47] Gurobi Optimization, LLC, "Gurobi optimizer reference manual," 2020.



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