LINEAR PROGRAMMING FOR STRATEGIC OPTIMIZATION OF SHARED AUTONOMOUS VEHICLE OPERATION AND INFRASTRUCTURE DESIGN

Cover Page

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Keywords: vehicle routing, ridesharing, fleet size, network design, parking space allocation

Subject area: (Please put an "X" as appropriate, you may choose more than one)

________1. dynamic route and/or departure time choice
________2. within-day dynamic equilibrium
________3. day-to-day dynamic processes
________4. dynamic network loading and traffic simulation (micro/meso/macro)
________5. dynamic link flow models and node models
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________7. dynamic OD estimation
___X___8. dynamic traffic control and management
________9. online dynamic models
________10. activity-based modeling and analysis
________11. travel behavior and travel choice principles
________12. mathematical foundations for networked ODE/PDE systems with routing functions at nodes.
________13. Control theoretic approaches to network flow dynamics
________14. game theory on networks, Nash-Stackelberg games
___X___15. dynamic network models for connected/automated vehicles
___X___16. dynamic network models for new mobility services
___X___17. dynamic network models for other emerging applications (smart transport, intelligent transportation systems, electric and green vehicles, public transport, bike repositioning, dynamic vehicle routing)
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Linear Programming for Strategic Optimization of Shared Autonomous Vehicle Operation and Infrastructure Design

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1 INTRODUCTION

Shared autonomous vehicle (SAV) systems can be an efficient transportation mode in the future (Fagnant and Kockelman, 2015). In an SAV system, autonomous vehicles shared by the society will transport travelers by using optimized routes and/or ridesharing matching. Thus, it will decrease number of vehicles and parking lots in a city without sacrificing travelers utility.

Design of SAV systems involve various types of problems. The notable examples are vehicle routing problem with pickup and delivery with time windows (VRPPDTW) (Mahmoudi and Zhou, 2016; Aiko *et al.*, 2017), dynamic ridesharing matching (Regue *et al.*, 2016; Aiko *et al.*, 2017; Thaithatkul *et al.*, 2019), fleet size optimization (Vazifeh *et al.*, 2018), road network design and autonomous vehicle lane deployment (Chen *et al.*, 2016), and parking space allocation. In the previous studies, these problems were often solved separately. Furthermore, they were often formulated by using computationally costly frameworks such as mixed integer programming.

This study proposes a unified optimization problem for aggregated versions of VRPPDTW, dynamic ridesharing matching, fleet size optimization, road network design, and parking space allocation of SAV systems based on dynamic traffic assignment (DTA). The proposed problem is formulated as linear programming, making it very easy to solve. Meanwhile, it approximates vehicles and travelers as continuum flow. These features will be useful for strategic optimization of SAV operation and infrastructure design.

2 FORMULATION

The proposed problem is based on the maximal flow problem with a time-expanded network. Specifically, a road network is modeled as a time-expanded network shown in Fig. 1. Then, vehicles and passenger flows, link capacity, and node capacity in the expanded network are optimized under given time-dependent origin-destination (OD) matrix of passengers.

The basic idea of the problem is as follows. Let x_{ij}^t be the total number of SAVs that travel from node i to j at time step t. Let $y_{s,ij}^{k,t}$ be the total number of travelers with certain properties (destination s, departure time k) that travel from node i to j at time step t. Since travelers need to ride SAVs to travel, condition $\sum_{s,k} y_{s,ij}^{k,t} \leq \rho x_{ij}^t$ must be satisfied, where ρ is a given passenger capacity of a SAV. Furthermore, x_{ij}^t must be smaller than the link capacity. Our objective is to find the most efficient $y_{s,ij}^{k,t}$ and x_{ij}^t and other decision variables under proper constraints including the passenger capacity and the link capacity.

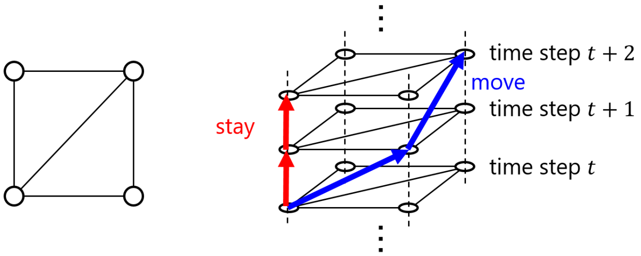

Figure 1: Left: standard network. Right: time-expanded network for dynamic traffic assignment

The problem is formulated follows:

$$\min N + \alpha \sum_{ij,s,t,k} y^{k,t}_{s,ij} + \beta \sum_{ij,i\neq j} x^t_{ij} \tag{1}$$

such that

$$\sum_j x^{t-1}_{ji} - \sum_j x^t_{ij} = 0 \qquad \forall i, t \in (0, t_{\max}) \qquad \text{(meaning: vehicle conservation)} \tag{2}$$

$$\sum_j y^{k,t-1}_{s,ji} - \sum_j y^{k,t}_{s,ij} + y^{k,t}_{s,0i} - y^{k,t}_{s,i0} = 0 \qquad \forall i, s, k, t \in T_k \qquad \text{(passenger conservation)} \tag{3}$$

$$\sum_{s,k} y^{k,t}_{s,ij} \leq \rho x^t_{ij} \qquad \forall ij, i \neq j, t \qquad \text{(passenger capacity of SAV)} \tag{4}$$

$$x^t_{ij} \leq \mu_{ij} \qquad \forall ij, i \neq j, t \qquad \text{(traffic capacity of link)} \tag{5}$$

$$x^t_{ii} \leq \kappa_i \qquad \forall i, t \qquad \text{(parking capacity of node)} \tag{6}$$

$$\sum_i x^0_{0i} \leq N \qquad \text{(fleet size)} \tag{7}$$

$$y^{k,k}_{s,0r} = M^k_{rs} \qquad \forall rs, k \qquad \text{(passenger departure)} \tag{8}$$

$$\sum_{t\in[k,k+d_{\max}]} y^{k,t}_{s,s0} = \sum_r M^k_{rs} \qquad \forall s, k \qquad \text{(passenger arrival)} \tag{9}$$

$$\sum_{ij} c_{ij}\mu_{ij} + \sum_i c_i\kappa_i \leq C \qquad \text{(construction budget)} \tag{10}$$

and some technical constraints such as non-negative constraints (omitted due to the space limitation), where μ_{ij} denotes the traffic capacity of link ij, κ_i denotes the parking capacity of node i, N denotes the total number of SAVs, M^k_{rs} denotes the time-dependent OD matrix, $d_{\max}$ denotes the maximum allowable delay for travelers, $t_{\max}$ denotes the final time step, c_{ij} denotes the unit cost for traffic capacity expansion of link ij, c_i denotes the unit cost for parking capacity expansion of node i, C denotes the construction budget, α and β denote weight parameters, and $T_k = \{t \in (0, t_{\max}) \cap (k, k + d_{\max}]\}$.

The objective function is a weighted sum of the SAV fleet size, total travel time of passengers, and total distance traveled by SAVs. Therefore, this problem is a system optimal DTA.

The decision variables are x^t_{ij} (corresponds to VRPPDTW with ridesharing), $y^{k,t}_{s,ij}$ (VRPPDTW with ridesharing), N (fleet size problem), μ_{ij} (link construction or SAV lane deployment problem), and κ_i (parking space allocation problem). Notice that these variables are in linear relationship in the problem. Thus, this is linear programming. The computation time is polynomial to the number of links and time steps.

This problem can be considered as a point queue-based DTA with vehicle queuing on nodes with limited queue length. The queue size on a node is constrained by (6); x^t_{ii} can be interpreted as the sum of parking vehicles and waiting vehicles on curbside. In fact, the problem can be considered as a variant of DTA-based optimal evacuation problem of Kuwahara *et al.* (2017). The limitation of this problem is that it only computes aggregated link flows; therefore, path flows and travel routes of individual travelers cannot be identified uniquely.

3 NUMERICAL EXAMPLES

Due to the space limitation, only VRPPDTW with ridesharing and fleet size optimization are considered in this abstract (numerical results on road network optimization and parking space allocation will be presented at the conference). Thus, the decision variables were limited to N, x_{ij}^{t}, and $y_{s,ij}^{k,t}$. Values of other parameters were fixed to represent a commute in a hypothetical one-dimensional city. The total number of travelers was 1000.

Scenarios with and without ridesharing ($\rho = 1$ or 2) were solved and illustrated in Fig. 2 as space–time diagrams. In the both scenarios, commuters were assumed to tend to travel from left to right following the same, given OD distribution. In Fig. 2a, SAV flows on many links were saturated (i.e., red colored flow in the right plot), and thus travelers needed to wait for long time at their origin nodes (i.e., thick vertical lines in the left plot). On the other hand, in Fig. 2b, SAV flow was uncongested and efficient traffic was realized because of ridesharing.

Table 1 shows efficiency of scenarios with fully privately-owned vehicles (this problem is formulated by slightly modifying the proposed problem), SAVs without ridesharing, and SAVs with ridesharing. According to the table, total number of vehicles required to serve the demand was greatly reduced by introduction of SAVs or ridesharing, and total distance traveled by vehicles was reduced by introduction of ridesharing. However, total travel time of passengers was increased due to detour of SAVs; regardless, the passenger was able to arrive the destination with acceptable delays as ensured by the arrival time constraint (9).

4 CONCLUDING REMARKS

Linear programming to jointly optimize SAV's routing and passenger pickup/delivery, passenger assignment and ridesharing, fleet size, road network design, and parking space allocation is developed based on a DTA framework. It can quantitatively determine when SAV and ridesharing are efficient from user and system perspectives. In the future, we will work on extension to multi-objective optimization problems, development of efficient solution algorithms, multi-modal transportation, passenger pricing, and application to real urban data.

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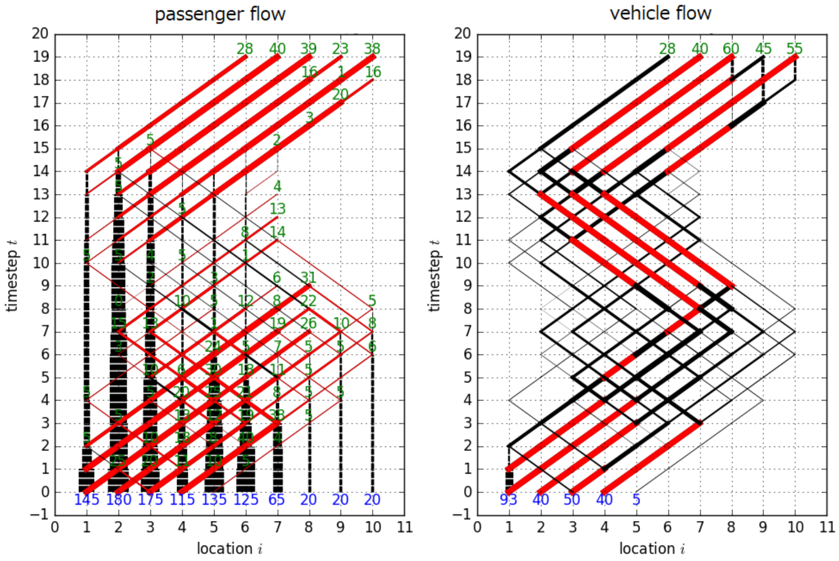
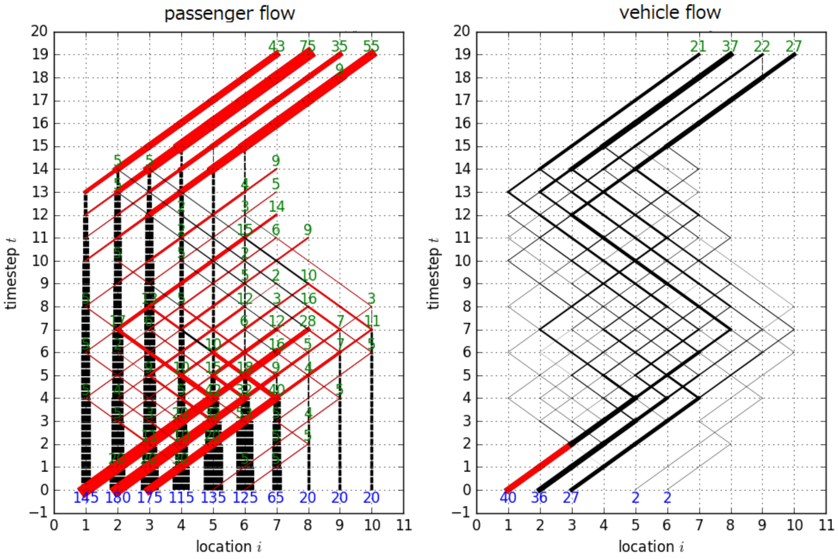
Table 1: Efficiency of different scenarios.

Scenario	Fleet size	Total travel time of travelers	Total distance traveled by vehicles
Fully privately-owned vehicles	1000	5500	3340
SAVs without ridesharing ($\rho = 1$)	228	8461	4184
SAVs with ridesharing ($\rho = 2$)	109	8388	2073



(a) Scenario without ridesharing ($\rho = 1$)



(b) Scenario with ridesharing up to two travelers ($\rho = 2$)

Figure 2: Space–time trajectories of travelers (left) and SAVs (right) in one-dimensional city. Width of each line represents traffic volume. Horizontal lines in the traveler flow represents waiting traveler at nodes. Red lines represent saturation flow. Blue numbers represent inflow, and green numbers represent outflow.

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