MODELING PEDESTRIAN FUNDAMENTAL DIAGRAM BASED ON DIRECTIONAL STATISTICS

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ABSTRACT

Understanding pedestrian dynamics is crucial for appropriately designing pedestrian spaces, such as corridors and public squares. The pedestrian fundamental diagram (FD), which describes the relationship between pedestrian flow and density within a given space, is crucial for characterizing these dynamics. Pedestrian FDs are strongly influenced by the direction of pedestrian flow, such as uni-directional, bi-directional, or multidirectional. Many researchers have analyzed pedestrian FDs with individual models for these specific situations. In this study, we propose a novel model for pedestrian FDs that can consider more general distribution of flow direction by using directional statistics, which is the statistics that deals with angular data. First, an indicator describing the pedestrian flow situation is developed solely from pedestrian trajectory data using directional statistics. Then, by incorporating this indicator into a traditional pedestrian FD model, we propose a new FD model to represent various pedestrian flow situations. We applied the proposed model to pedestrian trajectory data and validated its performance. The results confirm that the model effectively represents the essential nature of the multidirectional pedestrian flow, such as the capacity reduction depending on direction distribution.

Keywords: Pedestrian fundamental diagram, Directional statistics, Multidirectional pedestrian flow

1. INTRODUCTION

Pedestrian spaces like plazas and corridors have gained prominence in urban development. Evaluating pedestrian flow and performance of these spaces is critical. The Fundamental diagram (FD), which is the relation between flow and density initially used for vehicle traffic flow, is essential in understanding pedestrian flow characteristics and pedestrian space performance.

Numerous studies discuss pedestrian FDs, highlighting factors influencing them, including flow types like uni-directional, bi-directional, and crossing (Vanumu *et al.*, 2017). These flow types are illustrated in Figure 1. Observations suggest that bi-directional pedestrian flow has less flow than its uni-directional counterpart (Lam *et al.*, 2003). A pedestrian FD model considering bi-directional flow phenomena was also developed (Flötteröd and Lämmel, 2015). Still, the crossing effects of angled pedestrian flows in various directions remain underexplored. For instance, impact on pedestrian flows crossing at angles on FD is unclear (Cao *et al.*, 2017). In addition, in previous studies, different models were applied to different flow types. For instance, Fruin (1971) described FD for uni-directional and bi-directional flow with equations using different parameters. Consequently, to the authors' knowledge, no comprehensive pedestrian FD model exists for various flow types.

Modeling a comprehensive pedestrian FDs needs to incorporate the angles of pedestrian flow to capture the flow types such as bi-direction and crossing. However, incorporating angles is challenging due to their unique nature: periodicity. For example, the difference between 1 degree and 359 degree is smaller than that between 1 degree and 180 degrees. This is completely different from ordinally 1-dimensional numbers, so that ordinal statistics cannot be applied to angles. To account for this challenge, this study employs directional statistics: a statistics branch dealing with angles (Mardia and Jupp, 1999). Although it has been used in several transportation studies recently (Boeing, 2019, Nagasaki *et al.*, 2019, Nagasaki *et al.*, 2020), its application in pedestrian behavior is unseen.

The study's goal is to develop a pedestrian FD model applicable to various flow types, emphasizing angular statistics of pedestrian direction. To the authors' knowledge, this is the first attempt of its kind. Specifically, the study uses variables based on directional statistics as explanatory variables, representing the pedestrian flow angle's effect on FD. The model's validity is assessed using actual pedestrian trajectory data.

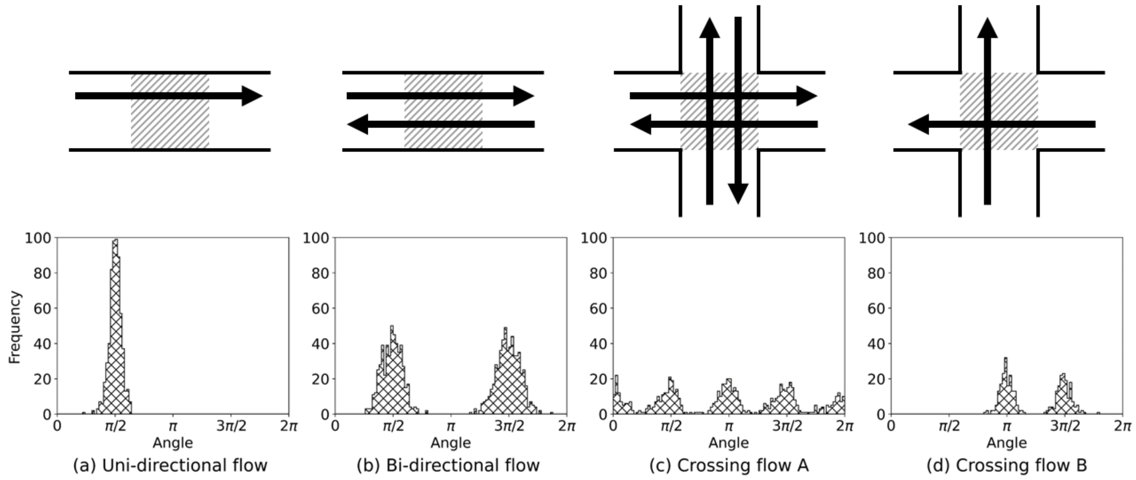

Figure 1. Examples of flow types

2. MODELING FD

2.1 Unique features of pedestrian flow

One of the unique features of pedestrian flow is that pedestrians move on a two-dimensional plane, and the moving direction of each pedestrian differs from each other. In this respect, pedestrian flow differs significantly from traffic flow, which flows in one-dimensional roads. Since FD was initially developed for traffic flow, a pedestrian FDs model must inherently accommodate this disparity.

An important factor in modeling pedestrian FDs is that pedestrians often conflict with the others' movement. In that case, pedestrians are forced to move in undesirable directions, and the flow decreases. Hence, in scenarios where pedestrian flows cross at various angles, the flow rate is anticipated to be lower than in the case of uni-directional pedestrian flows due to an increased potential for conflicts. This phenomenon has been verified in previous research; Zhang and Seyfried (2013) conducted experimental work demonstrating that the maximum flow of bi-directional flow is inferior to that of uni-directional flow.

Lane formation is another noteworthy phenomenon associated with pedestrian flows. Instances of lane formation have been documented in various experiments (Feliciani and Nishinari, 2016, Jin *et al.*, 2019). Furthermore, Lee *et al.* (2016) showcased, through simulation, an improved capacity resulting from the occurrence of lane formation. Therefore, bi-directional flows in which lane formation could occur are more efficient than those in which multiple flows cross at other angles, such as a crossroads.

In summary, the following two things can be noted about pedestrian flow. First, when flows cross at multiple angles, the flow decreases due to pedestrian conflicts. The second is that bi-directional flow is less likely to reduce the flow rate than when flows cross at other angles due to lane formation.

2.2 Angular variance

Angular variance is a statistic that describes the degree of dispersion for angular data. Angular variance v_1 for N angle data is expressed by the Eq. (1),

$$v_1 = 1 - \bar{R}_1, \tag{1}$$

where,

$$\bar{R}_1 = \sqrt{\bar{C_1}^2 + \bar{S_1}^2}, \tag{2}$$

$$\bar{C}_1 = \frac{1}{N}\sum_{j=1}^{N} \cos(\theta_j), \bar{S}_1 = \frac{1}{N}\sum_{j=1}^{N} \sin(\theta_j). \tag{3}$$

We explain the concept of angular variance v_1. For the ith angle data θ_i, $(\cos\theta_i, \sin\theta_i)^T$ represents the unit vector for that angle. Therefore, $(\bar{C}_1, \bar{S}_1)^T$ is the composite of N unit vectors divided by its number of samples N. $\bar{R}_1$, which is the norm of $(\bar{C}_1, \bar{S}_1)^T$, takes a value between 0 and 1, and it varies depending on the degree of dispersion of data. For example, $\bar{R}_1$ equals 1 when all data are toward the same angle, $\bar{R}_1$ equals 0 when all data are toward opposite. Therefore, angular variance $v_1 = 1 - \bar{R}_1$ represents the degree of dispersion of data.

However, angular variance cannot distinguish random direction data from data with multiple peaks in opposite directions. For example, random data and data peaked at $\pi/2$ and $3\pi/2$ both have angular variances close to 1 and cannot be distinguished.

Therefore, to distinguish such data, we define the pth-degree of angular variance v_p as Eq. (4), (5) and (6). That is equal to angular variance obtained after multiplying the angle data by p. This makes v_p small for an angle dataset with peaks every period $2\pi/p$[rad]. For example, for bi-directional flow, v_2 takes a small value because the direction of pedestrian movement has two peaks with period π.

$$v_p = 1 - \bar{R}_p, \tag{4}$$

$$\bar{R}_p = \sqrt{\bar{C_p}^2 + \bar{S_p}^2}, \tag{5}$$

$$\bar{C}_p = \frac{1}{N}\sum_{j=1}^{N} \cos(p\theta_j), \bar{S}_p = \frac{1}{N}\sum_{j=1}^{N} \sin(p\theta_j). \tag{6}$$

Examples of pth degree of angular variance are shown below. pth degree of angular variances were calculated for four different flow types, uni-directional flow, bi-directional flow, crossing flow A and crossing flow B. Uni/bi-directional flow means flow passing through a corridor uni/bi-directionally. Crossing flow A is defined as the case of bi-directional flow through a crossing, and crossing flow B is defined as the case of uni-directional flow through a crossing. Table 1 shows concepts of the four flow types and histograms of the pedestrian angles. The data used in this section are 10 seconds excerpt taken from the data used in the case study described in Chapter 3. Table 1 shows the value of pth degree of angular variance v_p $(p = 1,2,3,4)$ for each flow type. pth degree of angular variance could be confirmed to take small value when the angular data had peaks every $2\pi/p$[rad] period. Second degree of angular variance v_2 of bi-directional flow and fourth degree of angular variances v_4 of crossing flow A took small values. In addition, crossing flow B had a smaller angular variance v_1 than bi-directional flow and crossing flow A because the interval between peaks is narrow. In summary, uni-directional flow has small angular variance v_1 and small second degree of angular variance v_2, bi-directional flow has large v_1 and small v_2, crossing flow A has large v_1 and large v_2 and crossing flow B has relatively small v_1

and large ν_2. Therefore, the four flow types can be distinguished by angular variance ν_1 and second degree of angular variance ν_2.

Table 1. Angular variance of sample data

Flow type	ν_1	ν_2	ν_3	ν_4
Uni-directional	0.013	0.052	0.112	0.187
Bi-directional	0.958	0.166	0.975	0.514
Crossing A	0.958	0.947	0.931	0.459
Crossing B	0.303	0.944	0.407	0.245

2.3 Formulation of FD

2.3.1 Base function

One of the simple pedestrian FD models without the effect of the angles can be described as

$$J = \min(u\rho, C), \tag{7}$$

where J is the flow, u is the free flow velocity, ρ the density and C is the capacity. $u\rho < C$ in free flow regime and $u\rho > C$ in congested regime. The extreme congestion regime where flow decreases as density increases is omitted from Eq (7) because it will not be realized under ordinally pedestrian flow conditions.

Eq. (7) is not smooth because it uses minimum function. In order to smoothly connect free flow regime and congested regime, log-sum-exp (LSE) function defined as Eq. (8) is applied. LSE function gives a smooth approximation of maximum function. By reversing the signs, it can also approximate minimum function. Eq. (7) is converted into continuous function by applying LSE function as Eq (9).

$$\mathrm{LSE}(x_1, \dots, x_n) \equiv \log\left(\sum\nolimits_{i=1}^{n} \exp(x_i)\right) \approx \max(x_1, \dots, x_n). \tag{8}$$

$$J = -\mathrm{LSE}(-u\rho, -C) = -\log(\exp(-u\rho) + \exp(-C)). \tag{9}$$

2.3.2 Incorporating angles into FD function

In the following, angular variance is incorporated in the model to consider the impact of crossing pedestrian flows as described in Section 2.1. When pedestrian flows cross, the flow decreases due to pedestrian conflicts. Pedestrian conflicts are expected to occur primarily at high density, i.e., in congested regime. Therefore, the crossing is assumed to affect capacity C.

First, the impact of crossing is modeled. If the directions of pedestrians have single peak the impact of crossing seems to be small. On the other hand, if they are dispersing, the impact of crossing seems to be large. As described in section 2.2, angular variance ν_1 can represent the dispersion of angle data. Therefore, the magnitude of the capacity C reduction due to crossing is controlled by angular variance ν_1 in the proposed model.

Next, the influence of lane formation is modeled. Lane formation occurs in bi-directional flow. For bi-directional flow, the second degree of angular variance ν_2 takes a small value because the direction of pedestrian movement has two peaks with period π[rad]. Since the capacity C is expected to increase when lane formation occurs, ν_2 is added to the model to represent this.

Additionally, the model incorporates the impact of surrounding walls within the observation area. The underlying assumption of the model is that an increased presence of walls corresponds to a heightened frequency of conflicts with walls, thereby resulting in a reduced capacity. To quantify the impact of walls, a metric termed the, a metric termed the "wall ratio" r is defined as Eq. (10), where L represents the total perimeter of the measurement area and l denotes the length of the passable pedestrian section within it. For examples, when square measurement areas are assumed, r equals to 0.5 for a corridor and r equals to 0 for a crossing. In the model, the capacity C is assumed to decrease with wall ratio r increasing.

$$r = \frac{L - l}{L} \tag{10}$$

Based on the discussion above, the capacity C is assumed to decrease due to angular variance ν_1, second degree of angular variance ν_2 and wall ratio r. Therefore, the capacity is expressed as Eq. (11), where C_0 represents the capacity without the influence of ν_1, ν_2 and r. γ_1, γ_2 and γ_{wall} are parameters.

$$C = C_0(1 - \gamma_1\nu_1 - \gamma_2\nu_2)(1 - \gamma_{\text{wall}}r) \tag{11}$$

Finally, as shown in Eq. (12), substituting Eq. (11) into Eq. (9) completes the pedestrian FD model proposed in this study. Figure 2 shows a schematic diagram of the models described in this section.

$$J(\rho, \nu_1, \nu_2, r) = -\log\left(\exp(-u\rho) + \exp\left(-C_0(1 - \gamma_1\nu_1 - \gamma_2\nu_2)(1 - \gamma_{\text{wall}}r)\right)\right) \tag{12}$$

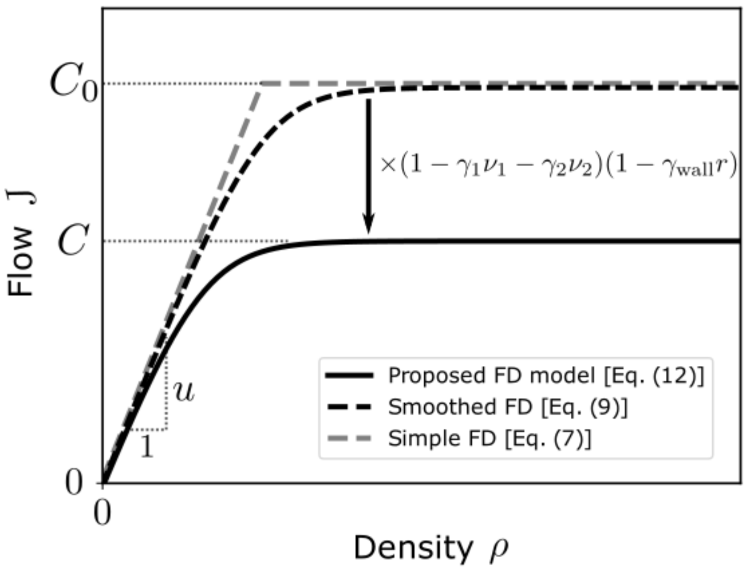

Figure 2. Schematic diagram of the models

3. CASE STUDY

3.1 Data

In this case study, "Corridor, unidirectional flow" dataset (DOI: 10.34735/ped.2013.6), "Corridor, bidirectional flow" dataset (DOI: 10.34735/ped.2013.5) and "Crossing, 90 degree angle" dataset (DOI: 10.34735/ped.2013.4) were used. They are provided by a German research institute, Forschungszentrum Jülich.

The pedestrian flows recorded in the dataset used in this study can be distinguished into four types, uni-directional flow, bi-directional flow, crossing flow A and crossing flow B. These flow types are defined in Section 2.2.

The flow J and density ρ are calculated from the trajectory data. The flow and density were obtained by applying the definition Edie (1963) as in Eq. (13) and (14), respectively. A is the spatio-temporal regime in which the flow rate and density are measured. The time region was set to 10 seconds, and the spatial region to square areas in the center of the corridor or crossing. d_s represents the distance that pedestrian s has traveled in spatio-temporal region A, and t_s represents the travel time in the region.

Figure 3 shows a scatter plot of the calculated flow J and density ρ. The capacity of the FD is different depending on the flow types. Therefore, explaining this data with a single equation is difficult with existing models.

$$J = \frac{\sum_s d_s}{A} \tag{13}$$

$$\rho = \frac{\sum_s t_s}{A} \tag{14}$$

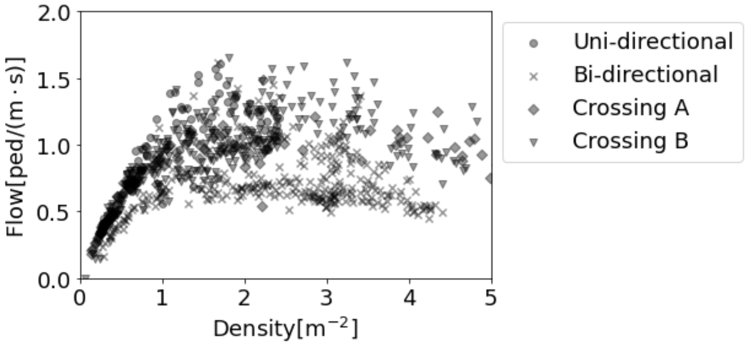

Figure 3. Density-Flow plots

In addition, angular variance ν_1, second degree of angular variance ν_2 and wall ratio r needed to be calculated to validate the model. ν_1 and ν_2 were calculated in the following way. First, the angles at which each pedestrian moved were acquired every 0.2 seconds or 0.25 seconds. Second, the angle data were compiled every 10 seconds. This time region is same as flow J and density ρ. Then, the angle variances were calculated by Eq. (3) and (6). Wall ratio r was calculated as defined in Eq (10).

80 data samples are taken for each flow type and 50 of them are train data and the other 30 are test data.

3.2 Parameter Estimation

The parameters of the model were estimated using the training data. Least squares method was applied to estimating. Next, the accuracy of the model was evaluated using test data.

Table 2 shows the parameter estimation results. The signs of the estimates of γ_1, γ_2, and γ_{wall} were positive. This is consistent with the assumption in Chapter 3 that the angular variance ν_1, the second degree of angular variance ν_2, and the wall ratio r decrease the capacity C. In addition, judging from the t-values and p-values, all parameters were significant. Table 3 shows the value of the coefficients of determination R^2 and adjusted coefficients of determination $\bar{R}^2$ for the training and test data, respectively. The coefficient of determination was approximately 0.6, indicating that the model had some goodness of fit.

Figure 4 shows the estimated FD. The projection of the estimated FD onto the flow-density plane can be drawn in numerous ways by changing variables ν_1, ν_2 and r. The FD projections representing to the four flow types are shown in Figure 4. Variables ν_1, ν_2 and r for uni-directional flow were set to $(\nu_1, \nu_2, r) = (0, 0, 0.5)$, for bi-directional flow were set to $(1, 0, 0.5)$, for crossing flow A were set to $(1, 1, 0)$ and for crossing flow B were set to $(0.3, 1, 0)$. The estimated FDs represent the variation of capacity due to flow types to some extent. In other words, this FD model could describe multiple flow types with a single function. This was not possible with existing models.

In bidirectional flow the estimated FD capacity appears to be higher than the data points especially at high density. This is because the second degree of angular variance ν_2 took large values at these data points due to collapse of lane formation.

Table 2. Estimation results

	Coefficient	t-value	p-value
u[m/s]	3.09	18.5	$< 10^{-10}$
C_0[ped/(m · s)]	1.53	15.2	$< 10^{-10}$
γ_1	0.224	6.20	3×10^{-9}
γ_2	0.210	3.37	9×10^{-4}
γ_{wall}	0.435	4.74	4×10^{-6}

Table 3. Coefficient of determination

	R^2	$\bar{R}^2$
Train data	0.654	0.646
Test data	0.615	0.599

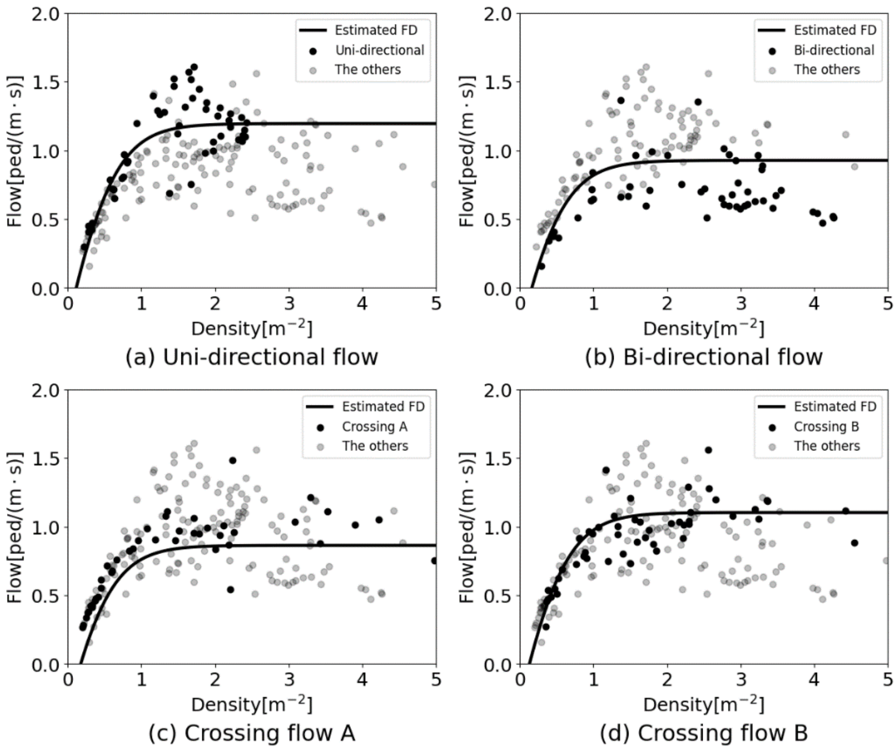

Figure 4. Estimated FD

4. CONCLUSIONS

We proposed a model of pedestrian FDs that can be applied to various types of uni/multidirectional flows. The original feature of the proposed FD is that it captures types of flow directions in a comprehensive manner (i.e., it does not require rule-based model switching as in existing studies). Specifically, we employ the angular variance to characterize the flow directions. It enables the FD to explain the effect of flow directions to flow performance, such as efficient bi-directional flows and inefficient crossing flows. The model was estimated and validated using actual pedestrian trajectory data.

The estimated values of the parameters were consistent with the assumptions of the modeling.

The results suggested that the complicated features of pedestrian flow due to the multi-directionality can be successfully represented by the directional statistics. The current model is still simple and has several limitations, as it is the first attempt of this kind. For example, it does not explain direction-dependent flow rate. In addition, it is questionable whether the model has the generality to be applicable to other flow types, such as T-junctions. Extension of the proposed model to incorporate such phenomena is considerable.

5. ACKNOLEDGEMENT

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